# Table of Contents

# Root class TDecompQRH -- how to use

## The issue

The question is actually: ***How to understand a completely misleading description of the class?***

The question has been raised in RootTalk -- TDecompQRH - how to use the householder decomposition? -- but was not answered properly by the author, Eddy Offermann .

## User's Guide on TDecompQRH

Here is a snpashot of the description from the Root User's Guide v5.26 (which differs very little from what can be found in Reference Manual or in the comments in source files):

**QRH**

Decompose a (*m* x *n*) - matrix *A* with *m* ≥ *n*.

```
A = QRH
```

*Q* orthogonal (*m* x *n*) - matrix, stored in `fQ`;

*R* upper triangular (*n* x *n*) - matrix, stored in `fR`;

**268** **Linear Algebra in ROOT**

*H* (*n* x *n*) - Householder matrix, stored through;

`fUp` n - vector with Householder up's;

`fW` n - vector with Householder beta's.

The decomposition fails if in the formation of reflectors a zero appears, i.e. singularity.

## Comments to the "guide"

Having inspected the code, I found that the code may be ok, but the description is wrong and requires the following corrections line by line:

| original | comment / corrected version |
|---|---|
| Decompose an (m x n)-matrix A with m ≥ n. | ok |
| A = QRH | A = QR<br>• no matrix H ! |
| Q : orthogonal (m x n) - matrix, stored in fQ; | **Everything is wrong!**<br>"Orthogonal" -- yes, but :<br>• (m x m) instead of (m x n)<br>• not stored in fQ<br>• Q is not explicitly stored anywhere<br>Q is defined as $Q = Q_1 Q_2 \cdots Q_t, \quad t = \min(m-1, n)$ , where |

| | |
|---|---|
| | $Q_k = \begin{pmatrix} I_{k-1} & 0 \\ 0 & Q'_k \end{pmatrix} \quad ,$<br><br>where $Q'_k$ is a *Householder matrix* of dimension $(m_k \text{ x } m_k)$ ( with $m_k = m - (k-1)$) which performs a *Householder transformation* by reflecting $\mathcal{R}^{m_k}$ about a hyperplane that is perpendicular to a *Householder vector* [1]<br><br>$v^{(k)} = (v_1^{(k)}, ..., v_{m_k}^{(k)})^T \quad .$ ( [2])<br><br>$Q'_k$ can be expessed via $v^{(k)}$ as<br><br>$Q'_k = I_{m_k} - 2\frac{(v^k)(v^k)^T}{(v^k)^T(v^k)} = I_{m_k} + \beta_k (v^k)(v^k)^T \quad ,$<br>where<br>$\beta_k = -2/((v^k)^T(v^k)) \quad .$<br>That allows ***the matrix Q to be stored implicitly*** via a set of $v^{(k)}$ (in fQ and fUp, see below) and -- for faster computations in future -- a set of $\beta_k$ (in fW).<br><br>Now we can describe ***what does fQ contain?***<br><br>• ***before*** the decomposition: it contains the matrix A<br>• ***after*** the decomposition:<br>♦ it contains the upper triangular matrix R in the diagonal and above the latter<br>♦ it also contains ***a part*** of vectors $v^{(k)}$, namely, the k-th column contains ***under the diagonal*** $(v_2^{(k)}, ..., v_{m_k}^{(k)})^T$<br><br>i.e. the $v^{(k)}$ without its first element |
| R : upper triangular (n x n)-matrix, stored in fR; | Right. R is found as $R = Q^T A = Q_t \cdots Q_2 Q_1 A$, calculated iteratively:<br>$A = Q_1 A, \quad A = Q_2 A, \ ... \ , \ R = Q_t * A$<br>(to be noted: $Q_i = Q_i^T = Q_i^{-1}$).<br>***First***, R is computed and placed into upper part of fQ, ***then*** copied into fR . |
| H : (n x n)-Householder matrix, stored through; | The H matrix is a **nonsense**: as we see, the involved matrices are $Q_i$ of dimension (m x m), and the matrices $Q'_i$ of dimensions (m x m), (m-1 x m-1), ... . |
| fUp : n-vector with Householder up s; | This is correct **if** one deciphers [3] the ***"Householder up s"*** as ***upper components of Householder vectors*** $v^{(k)}$, i.e.<br>$fUp = (v_1^{(1)}, v_1^{(2)}, \ ... \ , \ v_1^{(n)})^T$ |
| fW : n-vector with Householder beta s. | Similarly, an explicit definition is much better:<br>$(fW)_i = \beta_i \quad .$ |
| The decomposition fails if in the formation of reflectors a zero appears, i.e. singularity. | ? (I did not check this statement) |

## Footnotes

**[1] Definition of Householder vectors for the decomposition**.
The vector $v = v^{(1)}$ is chosen such that the corresponding Householder transform would reflect the first column of matrix A

$$x = (x_1, x_2, \ \dots, x_m)^T = (A_{11}, \ \dots, A_{m1})^T$$

onto the vector $e_1 = (1, 0, \ \dots, 0)^T$. This ensures that $A' = Q_1 A$ will have zeros in the the first column under the diagonal. It is quite obvious that there are two options: $x$ can be reflected

- either into $|x| \ e_1$ with $v = x - |x| \ e_1$
- or into $-|x| \ e_1$ with $v = x - (-|x|)e_1$

The choice is made in favour of $v = x + \text{sign}(x1) \ |x| \ e_1$ in order to avoid the $|v| \approx 0$ when $x$ is almost collinear with $e_1$ (in the latter case, an equality $|x_1| \approx |x|$ may lead to a big loss of precision while calculating $v_1 = x_1 - |x| \ \text{sign}(x_1)$). Then,

$$v = (x_1 + |x| \ \text{sign}(x_1), x_2, \ \dots, x_m)^T \quad .$$

The next Householder vector, $v = v^{(2)}$, is defined similary via the second column of matrix $A' = Q_1 A$, such that $A^{(2)} = Q_2 A'$ would have zeros under the diagonal in the 2nd column, and so on.

**[2] Actually, matrices $Q_k$ also represent Householder reflections** in $\mathcal{R}^m$ with *Householder vectors*

$$v'^{(k)} = (\underbrace{0, \ \dots, 0}_{\text{(k-1) times}}, v_1^{(k)}, \ \dots, v_{m_k}^{(k)})^T$$

**[3] Do terms "Householder's up's and beta's" come from the book "Matrix Computations" by Golub and Van Loan ?**

There is a comment in the text of `TDecompQRH::Decompose` function:

- *"QR decomposition of matrix a by Householder transformations,*
  *see Golub & Loan first edition p41 & Sec 6.2."*

The terms "up's and beta's" may be used in this book.
It looks a bit strange to refer to the ***first*** edition for the code written at ~2004, i.e. 8 years after the ***third*** edition had been published.
I did not find the 1ed, but found 3ed (here) and 2ed (in russian, 8MB or 58MB). In those editions, the notion of *Householder's up's* seems to be absent. Also the term *Householders beta's* is not used, though the symbol $\beta$ is employed for what is called *Householders beta* above, sometimes with an opposite sign.

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This topic: Main > AVFedotovHowToRootTDecompQRH
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