Glauber Monte Carlo results for nucleus-nucleus and proton-nucleus collisions at the Large Hadron Collider and the role of the neutron skin

David d'Enterria[1] and Klaus Reygers[2]

[1]*ICREA & ICC-UB, Univ. de Barcelona, 08028 Barcelona, Catalonia*
CERN, PH Department, 1211 Geneva, Switzerland
[2]*Physikalisches Institut, Univ. Heidelberg, Philosophenweg 12, 69120 Heidelberg, Germany*

(Dated: January 10, 2011)

The spatial distribution of nucleon-nucleon interactions in collisions involving heavy nuclei at CERN LHC energies is discussed using a Glauber Monte Carlo model. We parameterize the nuclear transverse density in terms of a standard single Fermi distribution for the nucleons as well as of two independent Fermi distributions for protons and neutrons to account for their different densities close to the nuclear periphery. The number of participant nucleons, binary nucleon-nucleon collisions, overlap area, eccentricity and average path-length of the interaction region, are presented with associated uncertainties as a function of impact parameter and of percentile centrality for lead-lead (Pb-Pb) and proton-lead (p-Pb) collisions at center-of-mass energies of relevance for the LHC heavy-ion programme, $\sqrt{s_{NN}}$ = 2.76 – 8.8 TeV. The total p-Pb and Pb-Pb geometric cross sections are also computed. The inclusion of separated proton and neutron transverse distributions to account for the neutron skin of the lead nucleus, does not alter significantly the values of all relevant quantities.

PACS numbers:

I. INTRODUCTION

The interpretation of many results measured in high-energy heavy ion collisions relies on the use of a model of the initial matter distribution resulting from the overlap of the two colliding nuclei at an impact parameter b. Indeed, quantities such as (i) the centrality dependence of any observable, proxyed via the *number of participating nucleons* in the collision $N_{\rm part}(b)$, (ii) the nuclear modification factor ($R_{\rm AA}$) obtained from the ratio of A-A over p-p spectra appropiately scaled by the nuclear *overlap function* $T_{\rm AA}(b)$ or the *number of independent nucleon-nucleon collisions* $N_{\rm coll}(b)$, (iii) the elliptic-flow parameter v_2 normalized by the *eccentricity* of the overlap region ε, and the average (iv) *path-length* $L(b)$ and (v) surface $A_{\rm T}(b)$ of the interaction region; depend all on a realistic model of the collision geometry.

The standard method employed in high-energy heavy ion collisions, relies on the description of the initial transverse shape of the nuclei in terms of 2-parameter Fermi (2pF) distributions with half-density radius R and diffusitivity a parameters obtained from fits to elastic lepton-nucleus data [1, 2], complemented with a Glauber eikonal model [3] to determine the underlying multi-nucleon interactions in the overlap area between the nuclei. In the Glauber Monte Carlo approaches (see [4] for a recent review), individual nucleons are sampled event-by-event from the underlying 2pF distribution and the collision properties are calculated by averaging over multiple events. It is known, however, that the proton and neutron distributions may not be exactly the same at the surface of heavy stable nuclei [5]. Such an effect is particularly important in neutron-rich nuclei, such as ^{208}Pb with an $N/Z \approx 1.54$ neutron excess, where the large Coulomb barrier reduces the proton density at the surface of the nucleus and concurrently "pushes out" the neutrons against the surface tension generated by the nuclear mean-fied. . As a result, the neutron can have either a "skin-type" distribution with 2pF half-density radius larger than that of the proton ($R_n > R_p$) but equal diffuseness parameter ($a_n = a_p$), or a "halo-type" distribution with $R_n = R_p$ and $a_n > a_p$ [6].

In this paper, we present the results of a Glauber Monte Carlo calculation for Pb-Pb and p-Pb collisions at LHC energies, $\sqrt{s_{NN}}$ = 2.76, 3.15, 5.5 and 8.8 TeV, that considers identical as well as separated transverse profiles for protons and neutrons in the lead nucleus. Section II summarizes the basic quantities of interest computed in the paper. In Section III we derive a parametrization of the center-of-mass energy dependence of the nucleon inelastic cross section (σ_{NN}), which is one of the key ingredients of the approach, based on the existing proton-proton (p-p) and proton-antiproton (p-$\bar{p}$) data. Section IV discusses the two different implementations of the nuclear transverse profile considered, based either on a single 2pF nucleon distribution or in separated 2pF's for protons and neutrons. In Section V we present the obtained results for T_{AB}, $N_{\rm coll}$, $N_{\rm part}$, ε, ε_{part}, and L as a function of the centrality $\mathscr{C}$ and impact parameter b for Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76, 3.15, 5.5 TeV, as well as for p-Pb collisions at $\sqrt{s_{NN}}$ = 2.76, 5.5, 8.8 TeV. The center-of-mass (c.m.) energies chosen correspond to those effectively operated at the collider

($\sqrt{s_{NN}}$ = 2.76 in 2010) or expected to be operated (in 2011-2012[1], and beyond[2]) in the heavy-ion runs at the LHC. We summarize our conclusions in the last section of the paper.

II. GLAUBER FORMALISM

The standard procedure to determine the transverse overlap area, and other derived quantities in a generic proton-nucleus or nucleus-nucleus collision (A-B) separated by impact parameter b, is based on a simple Glauber multi-scattering eikonal model that assumes straight-line trajectories of the colliding nucleus constituents. A recent review that describes the basic formalism can be found in [4]. We recollect here the main formulas of the model used in our calculations.

A. Glauber MC quantities

a. Thickness and overlap functions: The primary quantity of a Glauber approach is the *thickness function* of the nucleus which gives the density of nucleons ρ per unit area $dx\,dy$ along the direction z separated from the center of the nucleus A by an impact parameter b, i.e.

$$T_A(b) = \int dz\, \rho_A(b,z) \quad \text{normalized so that} \int d^2b\; T_A(b) \;=\; A. \tag{1}$$

The *nuclear overlap function* of nuclei A and B colliding at impact parameter b, $T_{AB}(b)$, can be written as a convolution of the corresponding thickness functions of A and B over the element of overlapping area $d^2\vec{s}$ ($\vec{s} = (x,y)$ is a 2-D vector in the transverse plane, and $\vec{b}$ is the impact parameter between the centers of the nuclei):

$$T_{AB}(b) \;=\; \int d^2\vec{s}\; T_A(\vec{s})\; T_B(|\vec{b}-\vec{s}|) \quad \text{normalized so that} \int d^2b\; T_{AB}(b) = A\,B. \tag{2}$$

In *minimum bias* reactions, the *average* nuclear thickness (for proton-nucleus, p-A) and nuclear overlap functions (for A-B) read:

$$\langle T_A\rangle_{MB} \equiv \frac{\int d^2b\; T_A}{\int d^2b} \;=\; \frac{A}{\pi\, R_A^2} = \frac{A}{\sigma_{pA}^{geo}} \quad \text{and} \quad \langle T_{AB}\rangle_{MB} \equiv \frac{\int d^2b\; T_{AB}(b)}{\int d^2b} \;=\; \frac{A\cdot B}{\pi(R_A+R_B)^2} = \frac{AB}{\sigma_{AB}^{geo}} \tag{3}$$

b. Number of participating nucleons: The number of nucleons in the target and projectile nuclei that interact in a collision at impact parameter b is called the number of participants (or number of "wounded nucleons") and is given by [10, 11]

$$N_{\rm part}\left(\vec{b}\right) = A\int \hat{T}_A\left(\vec{s}\right)\left\{1-\left[1-\hat{T}_B\left(\vec{s}-\vec{b}\right)\sigma_{NN}\right]^B\right\}d^2s \;+\; B\int \hat{T}_B\left(\vec{s}-\vec{b}\right)\left\{1-\left[1-\hat{T}_A\left(\vec{s}\right)\sigma_{NN}\right]^A\right\}d^2s, \tag{4}$$

c. Number of binary nucleon-nucleon collisions: For a given nucleon-nucleon cross section σ_{NN}, one defines the number of binary NN collisions in a A-B collision at impact parameter b as

$$\begin{aligned} N_{coll}(x,y;b) \;&=\; \sigma_{NN}\; T_A\left(x+b/2,y\right) T_B\left(x-b/2,y\right)\;, \\ N_{coll}(b) \;&=\; \int dx\,dy\; N_{coll}(x,y;b) \;=\; \sigma_{NN}\; T_{AB}(b). \end{aligned} \tag{5}$$

From this last expression one can see that the nuclear overlap function, $T_{AB}(b) = N_{coll}(b)/\sigma_{NN}$ [mb^{-1}], can be thought of as the nucleon-nucleon luminosity (reaction rate per unit cross-section) in a AB collision at a given impact parameter b.

---

[1] The $\sqrt{s}$ = 8 TeV expected for the coming p-p run(s) corresponds to a nucleon-nucleon c.m. energy $\sqrt{s_{NN}}$ = (Z/A) $\sqrt{s}$ = 3.15 TeV in Pb-Pb.
[2] Where the nominal c.m. energies are expected to be reached: $\sqrt{s_{NN}}$ = 5.5 TeV and 8.8 TeV for Pb-Pb and p-Pb respectively.

d. Eccentricity of the interaction region: The eccentricity of a A-B collision at impact parameter b can be obtained from the asymmetry ratio between the x and y "semi-axis" dimensions of the overlap zone, weighted by the number of nucleon-nucleon collisions at b:

$$\varepsilon(b) = \frac{\left\langle y^2 - x^2 \right\rangle}{\left\langle y^2 + x^2 \right\rangle} \tag{6}$$

where one calculates the moments of the participant nucleons either with the x axis oriented along the nominal reaction plane (estimated using spectator nucleons) or along the short principal axis of the participant distribution itself ε_{part} [22**?**].

e. Overlap area of the interaction region: The effective transverse overlap area between the two nuclei can be defined as in [**?**]:

$$A_{\perp}(b) = 4\pi \sqrt{\langle x^2 \rangle} \sqrt{\langle y^2 \rangle}, \tag{7}$$

where the weighted averages are the same as in Eq. (6). We note that there is no commonly accepted definition of the absolute normalisation of the overlap area. Our area definition with maximum magnitude 4π, is four times larger than that defined in [**?**] but coincides practically with the geometrical overlap area of two disks with a uniform two-dimensional distribution of density.

f. Average path-length: The average path-length ...

$$L(b) = ..., \tag{8}$$

where ...

g. Geometric cross section: The inelastic cross-sections of a p-A or A-B reaction, can be obtained from the corresponding inelastic nucleon-nucleon NN cross-section, $\sigma_{_{NN}}(\sqrt{s})$ at the center-of-mass energy $\sqrt{s}$, via

$$\sigma_{pA}(\sqrt{s_{_{NN}}}) = \int d^2b \left[1 - e^{-\sigma_{_{NN}}(\sqrt{s})\, T_A(b)}\right] \text{, and } \sigma_{_{AB}}(\sqrt{s_{_{NN}}}) = \int d^2b \left[1 - e^{-\sigma_{_{NN}}(\sqrt{s})\, T_{AB}(b)}\right] , \tag{9}$$

B. Centrality dependence

Experimentally one uses the "reaction centrality" $\mathscr{C}$ as a proxy for the impact-parameter b of a given nucleus-nucleus collision, by dividing the particle production cross section into centrality bins $\mathscr{C}_k = \mathscr{C}_1, \mathscr{C}_2, \cdots$ according to some fractional interval $\Delta\mathscr{C}$ of the total cross section, e.g. $\Delta\mathscr{C} = 0.0-0.1$ represents the 10% most central collisions. The fraction of the geometric cross-section within impact parameters $b_1 < b < b_2$ is:

$$f_{geo}(b_1 < b < b_2) \; = \; \left[2\pi \int_{b_1}^{b_2} bdb \left(1 - e^{-\sigma_{_{NN}} T_{AB}(b)}\right)\right] / \sigma_{_{AB}}^{geo} , \tag{10}$$

which simply corresponds to a given centrality percentile $\Delta\mathscr{C}$ between 0% (most central) and 100% (most peripheral).

III. INELASTIC NUCLEON-NUCLEON CROSS SECTION

A crucial ingredient of any Glauber calculation is the inclusive inelastic nucleon-nucleon cross section, $\sigma_{_{NN}}$, at the same c.m. energy $\sqrt{s_{_{NN}}}$ of the nucleus-nucleus collision. The value $\sigma_{_{NN}}$ includes particle production contributions from both the hard parton-parton scatterings, computable above a given p_T cutoff from first-principles QCD, as well as from softer "peripheral" nucleon-nucleon interactions with a scale not very far from $\Lambda_{QCD} \approx 0.2$ GeV. In addition, diffractive particle production (from single and double diffractive scatterings as well as double-Pomeron exchange) constitutes also a significant fraction, up to 30% [12], of $\sigma_{_{NN}}$ at the collider energies of interest here. We note that at high c.m. energies, above a few tens of GeV, p-p and p-$\overline{p}$ (as well as n-n and n-p) collisions have all the same total and inelastic cross sections, as the differences in their valence-quark structure are less and less relevant. Since there is no experimental meausurement yet of the inelastic cross section beyond Tevatron energies, we have determined the values of σ_{NN} needed for our Glauber calculations in the range $\sqrt{s_{NN}}$ = 2.76 – 8.8 TeV from the substraction $\sigma_{inel}(\sqrt{s}) = \sigma_{tot}(\sqrt{s}) - \sigma_{el}(\sqrt{s})$ where $\sigma_{tot}(\sqrt{s})$ is parametrized by a 6-parameter theoretical Ansatz proposed by the COMPETE collaboration [14]

$$\sigma_{tot}(\sqrt{s}) = Z + B\,\ln^2(s/s_0) + Y_1\,s^{-\eta_1} - Y_2\,s^{-\eta_2}\,, \quad (11)$$

where the first term is sometimes referred to as the "Pomeron", the squared logarithm encodes the asymptotic $\sqrt{s}$-dependence which saturates the Froissart bound [13], and the last two terms reflect the exchange of different Regge trajectories in different processes. Such an expression reproduces well the world-data on total p-p and p-$\bar{p}$ cross-sections[3] as a function of the squared c.m. energy s [15]. For $\sigma_{tot}(\sqrt{s})$ we have directly used the parameters and uncertainties obtained by COMPETE in their best fit (RRPL2u21) [14, 15], namely $Z = 35.50 \pm 0.47$ mb, $B = 0.3076 \pm 0.098$ mb, $s_0 = 29.20 \pm 5.376$ GeV^2, $Y_1 = 42.59 \pm 1.354$ mb, $\eta_1 = 0.460 \pm 0.016$, $Y_2 = 33.36 \pm 1.039$ mb, and $\eta_2 = 0.5454 \pm 0.0068$.

For $\sigma_{el}(\sqrt{s})$ we have used a similar formula to Eq. (11) but with two parameters less

$$\sigma_{el}(\sqrt{s}) = Z_{el} + B_{el}\,\ln^2(s/s_0) + Y_{el}\,s^{-\eta_{el}}\,. \quad (12)$$

We have fitted the existing world elastic cross-section data [15] above $\sqrt{s}$ = 10 GeV, combining both p-p and p-$\bar{p}$ measurements and using the universal value s_0 = 29.20 GeV. The result of this fit procedure yields: $Z_{el} = 5.19 \pm 0.93$ mb, $B_{el} = 10.33 \pm 10.85$ mb, $Y_{el} = 0.090 \pm 0.0010$ mb, and $\eta_{el} = 0.415 \pm 0.275$, with a relatively large χ^2/dof = 142./62. due mostly to the different results measured by the E-710/E-811 and CDF experiments at the Tevatron (see below). In Fig. 1 we show the result of both parametrizations compared to the world total and elastic p-p and p-$\bar{p}$ cross section data points. Subtracting one fit from the other we obtain the $\sigma_{inel}(\sqrt{s})$ curve shown in solid black.

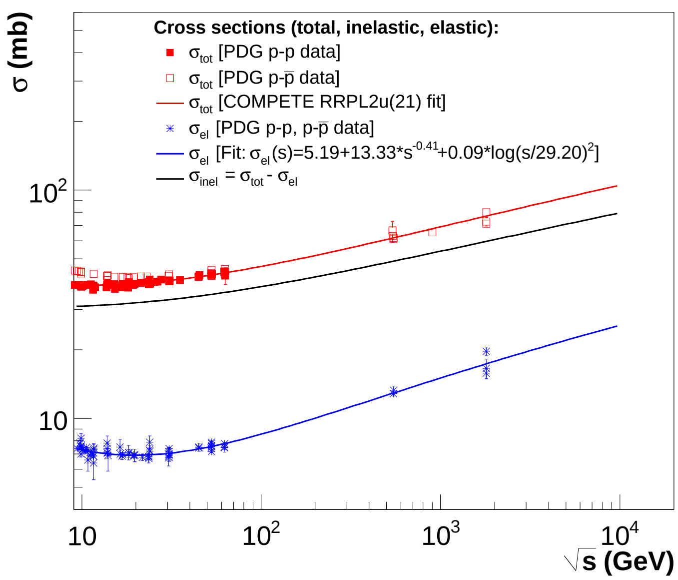

FIG. 1: World-data of total (squares) and elastic (star) cross sections in p-p and p-$\bar{p}$ collisions compared to Eq. (11) (red curve) and to Eq. (12) (blue curve) respectively. The black solid curve is the resulting inelastic cross section $\sigma_{inel}(\sqrt{s}) = \sigma_{tot}(\sqrt{s}) - \sigma_{el}(\sqrt{s})$.

TABLE I: Inelastic nucleon-nucleon cross sections, derived from the procedure discussed in the text, at the four different LHC energies considered in this work.

[3] The differences between the p-p and p-$\bar{p}$ cross-sections at low $\sqrt{s}$ are fully encoded in the Y_2 parameter, which has the same value but different sign for both systems.

$\sqrt{s_{NN}}$ (TeV)	2.76	3.15	5.5	8.8
σ_{NN} (mb)	64 ± 5	65 ± 5	71 ± 6	77 ± 6

The inelastic cross sections obtained by our procedure for the center-of-mass energies relevant for the heavy-ion and proton-nucleus programme at the LHC are listed in Table I. The obtained uncertainties are dominated by the different values of the experimental total and elastic cross sections at the highest c.m. energy measured so far: $\sigma_{tot}(\sqrt{s} = 1.8$ TeV$) = 80.03 \pm 2.24$ mb, 72.80 ± 3.10 mb, 71.42 ± 1.55 mb and $\sigma_{el}(\sqrt{s} = 1.8$ TeV$) = 19.70 \pm 0.85$ mb, 16.60 ± 1.60 mb, 15.79 ± 0.87 mb in p-$\bar{p}$ collisions at Tevatron by the CDF [16, 17], E-710 [18, 19] and E-811 [20, 21] experiments respectively.

IV. NUCLEAR SPATIAL TRANSVERSE DENSITIES

A. Single Fermi distribution

The matter distribution in transverse space of the colliding nuclei, $\rho(x, y)$ determines completely the nuclear thickness and overlap functions at impact parameter b and therefore all relevant quantities of interest presented in Section II A. The standard spatial density parametrization for nuclei is a two-parameter Fermi (2pF) distribution (also known as Woods-Saxon distribution) with radius R and surface thickness a

$$\rho(r) = \frac{\rho_0}{1 + \exp\left[(r - R)/a\right]}\,, \tag{13}$$

where ρ_0 is a normalisation constant so that $\int d^3r\, \rho(r) = 1$. The half-density or central radius R describes the mean location of the nucleus area (i.e., R is indicative of the extension of the bulk part of the density distribution) whereas a describes the diffuseness of the surface of the density profile.

The nuclear charge radius is nowadays known to within 1% for many nuclei thanks to the high accuracy achieved in elastic electron-nucleus and muon-nucleus scattering experiments [1, 2]. For the ^{208}Pb nucleus of interest in this work, its doubly-magic number of protons and neutrons ensures that deformations do not influence the results and a spherical density distribution describe the nuclear area very well. The experimental lepton-nucleus results in [1, 2] present the 2pF fit for ^{207}Pb (which we take, within uncertainties, to be that of ^{208}Pb too) with the R and a parameters listed in the first row of Table II.

TABLE II: Parameters R and a of the 2pF distributions for ^{208}Pb used in the present work. The single 2pF quoted in the first row is obtained from the experimental charge density measurements of [1, 2]. The parameters of the independent proton and neutron 2pF distributions are those obtained in [23] based on the experimental measurements of [**?**].

		R_q (fm)	a_q (fm)	$R_n - R_p$ (fm)	$a_n - a_p$ (fm)
nucleon		6.620± 0.006	0.546 ± 0.010	-	-
proton		6.704	0.438	-	-
neutron	“skin”-type	6.704	0.565	0.00	0.127
	“halo”-type	6.804	0.511	0.10	0.073

B. Double Fermi distribution (neutron “skin” and neutron “halo”)

Electromagnetic nuclear interactions [1, 2] – from which the nucleon parameters of the first row of Table II are extracted – are mostly sensitive to the *charged* (i.e. proton) distribution in the nucleus as well as, indirectly, to the neutron one because of its magnetic moment. It is known, however, that the proton and neutron distributions may not be exactly the same at the surface of stable nuclei. The effect is particularly large in neutron-rich nuclei, such as ^{208}Pb with an $N/Z \approx 1.54$ neutron excess, where the large Coulomb barrier reduces the proton density at the surface