Background-only hypothesis test with different methods for calculating significance

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Outline

• Methods for calculating significance
• Results
• Implementation example
• Conclusions
Methods

- Poisson distribution from data in signal region

\[ e^{-\mu} \frac{\mu^n}{n!} \]
Methods

• Poisson distribution from data in signal region

\[ e^{-\mu} \frac{\mu^n}{n!} \]

• Due to uncertainty in \( \mu = \mu_s + \mu_b \):
  multiply the Poisson with a Poisson/Gaussian from extra measurement

\[
e^{-\mu_s+\mu_b} (\mu_s + \mu_b)^n \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{(\mu_b-\langle \mu_b \rangle)^2}{2\sigma^2}} \times e^{-\mu_s+\mu_b} (\mu_s + \mu_b)^n e^{-\tau\mu_b} (\tau\mu_b)^{n_{off}} \frac{1}{n!} \times \frac{1}{n_{off}!}
\]

\( \mu_s \) is signal, \( \mu_b \) is background = nuisance parameter
Methods

- Poisson distribution from data in signal region
  \[ e^{-\mu} \frac{\mu^n}{n!} \]

- Due to uncertainty in \( \mu \): multiply the Poisson with a Poisson/Gaussian from extra measurement

\[
e^{-\mu_s + \mu_b} \left( \mu_s + \mu_b \right)^n \times \frac{1}{\sqrt{2\pi \sigma^2}} e^{\frac{(\mu_b - \langle \mu_b \rangle)^2}{2\sigma^2}} \times e^{-\mu_s + \mu_b} \left( \mu_s + \mu_b \right)^n \times e^{-\tau \mu_b} (\tau \mu_b)^{n_{off}}
\]

\( \mu_s \) is signal, \( \mu_b \) is background = nuisance parameter

- In order to get rid of nuisance parameters, can either integrate or minimize over them
Methods

- $Z_{Bi}$: Binomial: exact classical solution
Methods

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- Hybrid recipe with Integration:
  - $Z_{\Gamma}$
  - $Z_{N}$ - Using Cumulative Distribution Function
  - Using CreateIntegral
Problem with c.d.f

Before
Problem with c.d.f

Before

After
Problem with c.d.f

Before

After

• c.d.f for as if continuous poisson distribution = \( \int f(x)dx \)
• pvalue(n) = 1 – c.d.f(n)
• what is in RooStats now
Problem with c.d.f

- Before

  ![A RooPlot of "n"]

  - Projection of integral of p.d.f
  - \( n \) from 0 to 10

- After

  ![A RooPlot of "n"]

  - Projection of integral of p.d.f
  - \( n \) from 0 to 10

- c.d.f for discrete poisson distribution
  \[ = \sum f(n) \]

- pvalue\((n) = 1 – c.d.f(n - 1)\)
Methods

- $Z_{Bi}$: Binomial : exact frequentist solution

- Hybrid recipe with Integration:
  - $Z_{\Gamma}$
  - $Z_{N}$
    - Using CDF
    - Using CreateIntegral

- Likelihood profile = minimization :

  $Z_{PL Poisson}^{PL}$
  $Z_{PL Gaussian}^{PL}$
  - Using ProfileLikelihoodCalculator & MINUIT
## Results

### Significance Comparison

<table>
<thead>
<tr>
<th>$n_{\text{obs}}$</th>
<th>4.0</th>
<th>6.0</th>
<th>9.0</th>
<th>17.0</th>
<th>50.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_b$</td>
<td>1.0</td>
<td>1.3</td>
<td>3.8</td>
<td>3.8</td>
<td>27.5</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>0.477</td>
<td>0.3</td>
<td>0.9</td>
<td>0.6</td>
<td>3.71</td>
</tr>
<tr>
<td>$Z_b$</td>
<td>1.66</td>
<td>1.665</td>
<td>2.63</td>
<td>2.631</td>
<td>1.82</td>
</tr>
<tr>
<td>$Z_T$</td>
<td>1.66</td>
<td>1.666</td>
<td>2.63</td>
<td>2.688</td>
<td>1.82</td>
</tr>
<tr>
<td>$Z_N$</td>
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<td>1.873</td>
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<td>2.713</td>
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<tr>
<td>$Z_{\text{P}} \text{Gaussian}$</td>
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<td>2.013</td>
<td>2.83</td>
<td>2.831</td>
<td>2.02</td>
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<tr>
<td>$Z_{\text{P}} \text{Poisson}$</td>
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<td>1.948</td>
<td>2.82</td>
<td>2.816</td>
<td>1.99</td>
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</table>

### Significance Comparison

<table>
<thead>
<tr>
<th>$n_{\text{obs}}$</th>
<th>67.0</th>
<th>200.0</th>
<th>523.0</th>
<th>498428.0</th>
<th>2119449.0</th>
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<tbody>
<tr>
<td>$\mu_b$</td>
<td>30.0</td>
<td>100.0</td>
<td>388.6</td>
<td>493434.0</td>
<td>2109732.0</td>
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<td>$\sigma_b$</td>
<td>7.75</td>
<td>31.6</td>
<td>8.1</td>
<td>702.4</td>
<td>433.8</td>
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<tr>
<td>$Z_b$</td>
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<td>2.893</td>
<td>2.20</td>
<td>2.203</td>
<td>5.93</td>
</tr>
<tr>
<td>$Z_T$</td>
<td>2.89</td>
<td>3.087</td>
<td>2.20</td>
<td>1.202</td>
<td>5.93</td>
</tr>
<tr>
<td>$Z_N$</td>
<td>3.44</td>
<td>3.429</td>
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<td>2.900</td>
<td>5.93</td>
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<td>$Z_{\text{P}} \text{Poisson}$</td>
<td>3.04</td>
<td>3.042</td>
<td>2.38</td>
<td>2.384</td>
<td>5.95</td>
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</tbody>
</table>
Results

Very good agreement between Cousins et al. and our RooStats implementation
Conclusion

• Tested and implemented all methods described by Cousins et al. in Roostats

• Good agreement between their results and ours

• Simple code which could be used as short tutorial for Roostats and comparison between groups