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HistFactory Likelihood

1 The Likelihood Template

The likelihood (for one bin, or for the global counting analysis) is given by

$$\mathcal{L}(\mu, \alpha_i) = \prod_{m \in \text{bins}} \text{Pois}(n_m|\nu_m) \cdot G(L_0|L, \Delta_L) \prod_{i \in \text{Syst}}^{5} N(m_i|\alpha_i, 1) \tag{1}$$

where m is an index over the bins of the template histograms, i is an index over systematic effects, ν_m is the expected number of events in bin m given by

$$\nu_m = \sum_{j \in \text{Samp}} LF_j \eta_j(\boldsymbol{\alpha})\, \sigma_{jm}(\boldsymbol{\alpha}), \tag{2}$$

where F_j is a product of unconstrained normalization factors for sample j

$$F_j = \prod_{n \in \text{NormFact}_j} f_n \tag{3}$$

that include our parameter of interest $\mu = \sigma/\sigma_{SM}$. The term

$$\eta_j(\boldsymbol{\alpha}) = \prod_{i \in \text{OverallSys}} \eta_{ij}(\alpha_i) \tag{4}$$

is composed

We must incorporate correlations among the different sources of systematics and their correlated impact among different signal and background contributions. This is achieved in two steps. First, we group the sources of systematics, α, such that the corresponding variations in η are expected to be uncorrelated (eg jet energy scale, trigger efficiencies, etc.). Next, we vary the sources of the systematics by the $\pm 1\sigma$ variations and determine the effect $\eta_{ij}^{\pm}$ for each signal and background. A change in the source of the s^{th} systematic will cause a totally correlated variation among the samples (indexed by j). Thus we describe η_{ij} a piece-wise linear function of α_j

$$\eta_{ij}(\alpha_j) = \begin{cases} \eta_{ij}^0 + \alpha_j(\eta_{ij}^+ - \eta_{ij}^0) & \text{if } \alpha_i > 0 \\ \eta_{ij}^0 & \text{if } \alpha_i = 0 \\ \eta_{ij}^0 - \alpha_j(\eta_{ij}^- - \eta_{sj}^0) & \text{if } \alpha_i < 0\ . \end{cases} \tag{5}$$

$$\sigma_{jm}(\boldsymbol{\alpha}) = \prod_{i \in \text{OverallSys}} \sigma_{ijm}(\alpha_j) \tag{6}$$

$$\sigma_{ijm}(\alpha_j) = \begin{cases} \sigma_{ijm}^0 + \alpha_j(\sigma_{ijm}^+ - \sigma_{ijm}^0) & \text{if } \alpha_i > 0 \\ \sigma_{ijm}^0 & \text{if } \alpha_i = 0 \\ \sigma_{ijm}^0 - \alpha_j(\sigma_{ijm}^- - \sigma_{ijm}^0) & \text{if } \alpha_i < 0\ . \end{cases} \tag{7}$$

2 Implementation with HistFactory

Note, when using the `HistFactory` the production modes l and backgrounds j correspond to a single XML `Sample` element. The `HistoName` attribute inside each sample element specifies the histogram with the σ^0_{ijm}. The index $j =$' J' is set by the `Name` attribute of the `Sample` element (eg. `<Sample Name='J'>`). Between the open `<Sample>` and close `</Sample>` one can add

- An `OverallSys` element where the `Name='I'` attribute identifies which α_I is the source of the systematic and implies that the Gaussian constraint $N(m_I|\alpha_I, 1)$ is present. The `High` attribute corresponds to η^+_{IJ}, eg when the source of the systematic is at $+1\sigma$ and $\alpha_I = 1$. Similarly, the `Low` attribute corresponds to η^-_{IJ}, eg when the source of the systematic is at -1σ and $\alpha_I = -1$. The nominal value is $\eta^0_{IJ} = 1$ for the ovreall systematics. The distinction between the sign of the source α and the effect η allows one to have anti-correlated systematics. The `HistFactory` is able to deal with asymmetric uncertainties as well, by using a piece-wise linear interpolation for the $\alpha_I > 0$ and $\alpha_I < 0$ regions.

- A `NormFactor` element is used to introduce an overall constant factor into the expected number of events. In the example below, the term $\mu = \sigma/\sigma_{SM}$ corresponds to the line `<NormFactor Name='SigXsecOverSM'>`. In this case, the histograms were normalized to unity, so additional `NormFactor` elements were used to give the overall cross-sections σ_J.

- A `HistoSys` element is used to introduce shape systematics and the `HistoNameHigh` and `HistoNameLow` attributes have the variational histograms σ^+_{ijm} and σ^-_{ijm} corresponding to $\alpha_i = +1$ and $\alpha = -1$, respectively.

Below is an example XML file for the electron channel.

```
<!DOCTYPE Channel  SYSTEM 'HistFactorySchema.dtd'>
  <Channel Name="channelEle" InputFile="./data/central_Ele_5jet_inc_35invpb.root" HistoName="" >
    <!--<Data HistoName="data" HistoPath="" />-->
    <Sample Name="bbAtautau120" HistoPath="" NormalizeByTheory="True" HistoName="bbAtautau120All">
      <OverallSys Name="JES" High="1.05" Low="0.95"/>
      <OverallSys Name="EVTEFF" High="1.122" Low="0.878"/>
      <OverallSys Name="bbAtautau" High="1.15" Low="0.85"/>
      <NormFactor Name="NEle_bbAtautau120" Val=".83202" Low=".83202" High=".83202" Const="True" />
      <NormFactor Name="SigXsecOverSM" Val="0" Low="-10." High="30." Const="True" />
    </Sample>
    <Sample Name="Atautau120" HistoPath="" NormalizeByTheory="True" HistoName="Atautau120All">
      <OverallSys Name="JES" High="1.05" Low="0.95"/>
      <OverallSys Name="EVTEFF" High="1.122" Low="0.878"/>
      <OverallSys Name="Atautau" High="1.15" Low="0.85"/>
      <NormFactor Name="NEle_Atautau120" Val=".24224" Low=".24224" High=".24224" Const="True" />
      <NormFactor Name="SigXsecOverSM" Val="0" Low="-10." High="30." Const="True" />
    </Sample>
    <Sample Name="bbAtautau130" HistoPath="" NormalizeByTheory="True" HistoName="bbAtautau130All">
      <OverallSys Name="JES" High="1.05" Low="0.95"/>
      <OverallSys Name="EVTEFF" High="1.122" Low="0.878"/>
      <OverallSys Name="bbAtautau" High="1.15" Low="0.85"/>
      <NormFactor Name="NEle_bbAtautau130" Val=".01767" Low=".01767" High=".01767" Const="True" />
      <NormFactor Name="SigXsecOverSM" Val="0" Low="-10." High="30." Const="True" />
    </Sample>
    <Sample Name="Atautau130" HistoPath="" NormalizeByTheory="True" HistoName="Atautau130All">
      <OverallSys Name="JES" High="1.05" Low="0.95"/>
      <OverallSys Name="EVTEFF" High="1.122" Low="0.878"/>
      <OverallSys Name="Atautau" High="1.15" Low="0.85"/>
      <NormFactor Name="NEle_Atautau130" Val=".02441" Low=".02441" High=".02441" Const="True" />
      <NormFactor Name="SigXsecOverSM" Val="0" Low="-10." High="30." Const="True" />
    </Sample>
    <Sample Name="Ztautau" HistoPath="" NormalizeByTheory="True" HistoName="ZtautauAll">
      <OverallSys Name="JES" High="1.05" Low="0.95"/>
      <OverallSys Name="EVTEFF" High="1.122" Low="0.878"/>
      <OverallSys Name="Alpgen" High="1.131" Low="0.869"/>
      <OverallSys Name="Ztautau" High="1.15" Low="0.85"/>
      <NormFactor Name="NEle_Ztautau" Val="1.26818" Low="1.26818" High="1.26818" Const="True" />
    </Sample>
   <Sample Name="AddOn" HistoPath="" NormalizeByTheory="False" HistoName="AddOnAll">
      <OverallSys Name="AddOn" High="1.173" Low=".827"/>
      <NormFactor Name="NEle_AddOn" Val=".88267" Low=".88267" High=".88267" Const="True" />
    </Sample>
```

```
  <Sample Name="SameSign" HistoPath="" NormalizeByTheory="False" HistoName="SameSignAll">
    <OverallSys Name="SameSign" High="1.06828" Low=".93172"/>
    <NormFactor Name="NEle_SameSign" Val="4.00568" Low="4.00568" High="4.00568" Const="True" />
  </Sample>
  <Sample Name="Others" HistoPath="" NormalizeByTheory="True" HistoName="OthersAll">
    <OverallSys Name="JES" High="1.05" Low="0.95"/>
    <OverallSys Name="EVTEFF" High="1.122" Low="0.878"/>
    <OverallSys Name="QFAC" High="1.03" Low="0.97"/>
    <OverallSys Name="Alpgen" High="1.131" Low="0.869"/>
    <OverallSys Name="Others" High="1.15" Low="0.85"/>
    <NormFactor Name="NEle_Others" Val=".17949" Low=".17949" High=".17949" Const="True" />
  </Sample>
</Channel>
```

One can convert this Gaussian constraints into a Poisson/Gamma systematic by adding lines like

```
<ConstraintTerm Type="Gamma" RelativeUncertainty="0.1">JES</ConstraintTerm>
```

to the `Measurement` element. For example:

```
<Measurement Name="AllSYS" Lumi="35.2" LumiRelErr="0.11" BinLow="0" BinHigh="20" Mode="comb"  ExportOnly="True">
  <POI>SigXsecOverSM</POI>
  <ParamSetting Const="True">NEle_AddOn,NEle_Atautau120,NEle_Atautau130,NEle_Others,
    NEle_SameSign,NEle_Ztautau,NEle_bbAtautau120,NEle_bbAtautau130,NMuo_AddOn,
    NMuo_Atautau120,NMuo_Atautau130,NMuo_Others,NMuo_SameSign,NMuo_Ztautau,NMuo_bbAtautau120,
    NMuo_bbAtautau130
  </ParamSetting>
  <ConstraintTerm Type="Gamma" RelativeUncertainty="0.1">JES</ConstraintTerm>
  <!--<ConstraintTerm Type="LogNormal" RelativeUncertainty="0.1">JES</ConstraintTerm>-->
</Measurement>
```