Initial Study of Hadronic Energy Resolution in the Analog HCAL and the Complete CALICE Setup

The CALICE Collaboration

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ABSTRACT: The hadronic energy resolution of the CALICE analog hadron calorimeter and of the complete CALICE setup, consisting of a SiW electromagnetic calorimeter, an analog scintillator-steel hadron calorimeter and an analog scintillator-steel tail catcher, has been studied using data taken in 2007 at CERN. To optimize the energy resolution for single hadrons by taking advantage of the high granularity and segmentation of these calorimeters, a simple signal weighting procedure based on the density of deposited energy in each cell is studied. These initial studies yield a stochastic term of the resolution of approximately $50\%/\sqrt{E}$ in the case of the HCAL stand alone and for the complete setup. This corresponds to an improvement of roughly 25% and 18% compared to the resolution obtained without density dependent weighting for the HCAL and the complete setup, respectively.

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1. Introduction

The CALICE collaboration has constructed highly granular calorimeter prototypes for future collider experiments. The CALICE calorimeters were tested in various different configurations in test beams at DESY, CERN and FNAL. For the data set studied in this note, taken in 2007 at CERN, a silicon-tungsten electromagnetic sampling calorimeter (ECAL) [1], a scintillator-steel hadron sampling calorimeter with analog readout (AHCAL) [2] and a scintillator-steel tail catcher and muon tracker (TCMT) [3] were installed.
The ECAL has a total depth of 24 radiation lengths $X_0$ and consists of 30 active layers arranged in three longitudinal sections with different samplings. The first 10 layers use 1.4 mm thick tungsten absorber plates (0.4 $X_0$), followed by 10 layers of 2.8 mm thick absorbers (0.8 $X_0$) and 10 layers of 4.2 mm thickness (1.2 $X_0$). The total thickness of the calorimeter is 20 cm. Each silicon layer has an active area of $18 \times 18$ cm$^2$, segmented into individual modules with $6 \times 6$ readout pads with a size of $1 \times 1$ cm$^2$. This results in a total of 9720 channels for the detector.

The AHCAL consists of small plastic scintillator tiles with individual readout by silicon photomultipliers (SiPMs) arranged in layers between 2 cm thick stainless steel absorber plates. The size of the scintillator tiles ranges from $3 \times 3$ cm$^2$ in the center of the detector to $12 \times 12$ cm$^2$ on the outer edges of the calorimeter. In the last eight layers only tiles with $6 \times 6$ cm$^2$ and $12 \times 12$ cm$^2$ are used. In total, the hadron calorimeter has 38 sensitive layers, amounting to a depth of 4.5 hadronic interaction lengths $\lambda_I$. The total number of scintillator cells is 7608.

The TCMT consists of 16 readout layers each with 20 $100 \times 5$ cm$^2$ scintillator strips read out by SiPMs between steel absorber plates, resulting in 320 readout channels. The detector is subdivided into a fine and a coarse section, where the first 8 layers have 19 mm thick absorber plates, while the absorbers for the last 8 layers are 102 mm thick. The orientation of the scintillator strips alternates between horizontal and vertical in adjacent layers. In total, the TCMT thickness corresponds to a depth of 5.8 $\lambda_I$.

This gives a total depth of approximately 11 $\lambda_I$ for the complete CALICE setup, and a total of 17648 readout channels. The unprecedented granularity of the CALICE calorimeters is reflected in this high number of readout channels for the test beam prototype. This compares to approximately 6000 cells for the complete ZEUS calorimeters.

The present note describes an analysis of hadron data taken at CERN in 2007 in which the hadronic energy resolution of the AHCAL and of the combined CALICE setup was studied. A simple signal weighting algorithm based on the local energy density in a calorimeter shower was developed and used to improve the measured energy resolution - both for showers contained wholly within the AHCAL and for showers in the complete CALICE setup. A preliminary version of the offline reconstruction was used in the analysis which did not include all known correction procedures - most notably absent are a cell-by-cell temperature correction for the SiPM readout and saturation correction using in-situ measurements of the saturation measurements for each cell. Further detail on the used reconstruction software can be found in Appendix.

A first study of the combined response of the CALICE setup, performed with 2006 data, can be found in.

2. Energy Reconstruction for Hadronic Showers and Weighting Techniques

For electromagnetic showers in a sampling calorimeter the energy seen in the active media is directly proportional to the incoming particle energy. The measured energy is the true particle energy modified by the constant sampling fraction for all energies, unless there are limitations in, e.g., readout electronics saturating at high signal levels. The calorimetric response to hadrons is much more complicated due to the more complex nature of hadronic interactions in material. These include a mixture of both hadronic and electromagnetic showers, invisible energy in the form of binding energy, nuclear recoil, and neutrinos, and (mostly unseen) energy in the form of low energy.
neutrons. The electromagnetic components stem from neutral pions created in the hadronic cascade, and is most prominent in the core of the shower. The average fractions of these components are of similar order in size and have varying energy dependencies. As a consequence, the observed signal for a given energy deposit is in general larger in the case of electromagnetic than in hadronic showers, commonly expressed as the ratio \( e/h > 1 \). The average electromagnetic fraction of a hadron shower increases with the energy of the incident particle, which in combination with the \( e/h \) value leads to a non-linearity in the detector response. Furthermore, for hadrons with a fixed energy, there are large fluctuations from event to event in the relative fractions of electromagnetic and hadronic subshowers. For a non-unity \( e/h \) this lead to deterioration of the energy resolution for hadrons.

Hadronic calorimeters which due to their construction have an \( e/h = 1 \) are called compensating calorimeters. Such detectors have been successfully used, for example in the ZEUS experiment [5]. Such calorimeters yield an improved hadronic energy resolution, however they are subject to very stringent requirements on the detector geometry and on the used materials.

Even in intrinsically non-compensating calorimeters, compensation can be achieved by identifying the nature of individual energy depositions (electromagnetic or hadronic) and by weighting them with a suitable factor in the reconstruction software. The identification of the nature of the energy deposit can be achieved via the local energy density in the detector, since electromagnetic showers tend to be much denser than purely hadronic showers. Such procedures have been successfully used for example in the ATLAS hadronic endcap calorimeter [6]. For this, longitudinal and lateral segmentation of the calorimeter readout is required.

Figure 1. Energy density for cells within showers initiated by 25 GeV positive pions, simulated with GEANT4 using the QGSP physics list.

Figure 1 shows the energy density for calorimeter cells in hadronic showers simulated with GEANT4 for 25 GeV positive pions. The detector model is a simplified version of the CALICE AHCAL, with alternating layers of 2 cm steel and 5 mm scintillator. The scintillator layers were
3. Analysis Overview

The analysis discussed in this note was performed on hadron data taken in 2007 at CERN with the SiW ECAL, the analog HCAL and the TCMT. The HCAL was configured in the not rotated position. The detailed run list used can be found in Appendix A. Only events with a valid beam trigger, defined by the coincidence of the small 10 × 10 cm² trigger scintillators and the spill signal, were analyzed.

After the reconstruction of the data, the deposited energy in each cell of the three detectors is available in units of MIPs, the most probable energy deposit by a through-going minimum ionizing particle. The calibration was performed with a muon beam at the beginning of the data taking period [7,8].

Two separate studies, based on the same principle, were performed, one for showers contained in the HCAL, and one for the complete CALICE setup, where no requirement on containment of the showers was made. Analysis details and results are given in Sections 4 and 5, respectively. The reconstructed energy and the corresponding energy resolution was studied for three separate cases:

- Energy independent single weights (one factor per detector). Here, one constant factor was applied per detector to convert the total energy in units of MIP to the energy scale in GeV.

- Individual weights depending on the local shower density, determined with a beam energy constraint for each data point. Here, the energy in each detector cell was weighted according to the energy content to convert from the MIP to the GeV scale. The set of weights were determined for each run, minimizing the energy resolution by using the known beam energy as a constraint.

- Energy dependent parametrization of weights depending on the local shower density. Also here, the energy in each detector cell was weighted according to the energy content. However, the weights that were used were determined from an energy dependent parametrization, thus not requiring the knowledge of the beam energy. The weights were selected using the energy reconstructed with the single weights method.
4. Hadronic Energy Resolution of the Analog HCAL

4.1 Event selection
To select a sample of showers fully contained in the AHCAL, a maximum energy deposit of 50 MIPs in the ECAL, 10 MIPs in the TCMT and a minimum energy deposit of 100 MIPs in the HCAL for each event are required. These cuts reject electrons as well as hadronic showers that start in the ECAL or leak into the TCMT. Furthermore, every hit has to be above a threshold of 0.5 MIPs. Isolated noise hits are rejected with an algorithm described in Section 4.2.1.

4.2 Analysis Technique

4.2.1 Noise Rejection
Since the method of weighting the energy deposit of individual cells according to the local energy density assigns higher weights to cells with low energy content, the influence of random noise hits right above threshold gets amplified in the total reconstructed energy sum. It is therefore desirable to remove such hits from the events if possible. For this analysis, this is achieved by rejecting hits that are isolated both laterally and transversely. This isolation is determined based on the total energy density in the immediate surrounding of the hit under study. For each detector channel, a three dimensional energy density, taking the energy deposits of all neighboring cells in three dimensions into account, was calculated. This energy density was given by the sum of the energy in the cell under study as well as all the energy in all neighboring cells in the same layer and in the layers before and after the hit. This density was not divided by the number of cells or by the volume. Instead, the total sum of the energy was used. In the determination of the neighboring cells the different cell sizes within the AHCAL were taken into account. As neighbors, both orthogonally and diagonally neighboring cells were used. Hits with a low value for this three dimensional density below 0.8 MIP were rejected. The cut value was determined using pedestal events. This rejection method reduced the contribution of noise to the overall signal considerably. However, in addition to rejecting random noise hits, this procedure also removes isolated neutrons. It was observed that this leads to a slight deterioration of the energy resolution when no energy density dependent weights are applied.

4.2.2 Energy Reconstruction with Single and Energy Density dependent Weights
The simplest way to calculate the reconstructed energy and the energy resolution of the AHCAL is to use one single conversion factor, or single weight, with which the sum of the MIP amplitudes of all cells is multiplied to get the reconstructed energy. An average weight factor of $c = 0.0305 \text{ GeV/MIP}$ was obtained, and was used for the single weight reconstruction of showers contained in the AHCAL.

To improve the energy resolution, a technique in which not one weight factor is used, but different weights for electromagnetic and hadronic sub-showers, was developed. The simplest way of implementing this is to distinguish between different types of energy deposit on the cell level using the local energy density in the shower, given by the energy deposited in that particular cell. Here, cells in electromagnetic sub-showers will typically have a higher energy deposit since the density in electromagnetic showers is higher than in hadronic showers. To develop this density
Figure 2. Energy density per detector cell in the AHCAL for negative pions at 20 GeV. The density is calculated relative to the volume of the $3 \times 3$ cm$^2$ cells. The subdivision of the energy density into ten different bins is illustrated by the color shading.

Dependent weighting, the individual energy deposits were subdivided into 10 different density bins. The total energy sum of each of these is then weighted with different factors $\omega_i$ ($i =$ index of the density bin). For the calculation of the energy density in a given cell, the MIP amplitude of that cell normalized to the cell volume is used, with the volume of the small $3 \times 3$ cm$^2$ taken as the base line. Thus the deposits in larger cells are divided by 4 or 16 to give the density, depending on the size of the cell. Figure 2 shows the energy density spectrum, divided into ten bins.

Suitable weights $\omega_i$ to minimize the energy resolution are found by minimization of the following function:

$$\chi^2 = \sum_{\text{events}} \left( \sum_i E_i \omega_i - E_{\text{beam}} \right)^2$$

This was done for all energies to have a set of $i$ weights for each run $\omega_i$ ($r = \text{run}$). In this determination, and also for the determination of the single weight factor discussed above, the energy loss of the incoming particle in the ECAL was taken into account by reducing the beam energy by 200 MeV. This corresponds to the mean energy loss of a minimum ionizing particle in this detector, calculated from the material properties.

With this method of using individual weights for each run, the energy was reconstructed and the energy resolution was calculated. Figure 3 shows the weights determined for showers of 20 GeV pions contained in the AHCAL. Fits to the weights with Equation 4.2 are shown for illustration.

It was found that the weights change with energy, as expected from the behavior of hadronic showers. To facilitate the parametrization of the energy dependence, the weights are parametrized by a function with 3 parameters, given by
Figure 3. Individual weights as a function of the energy density bin determined for 20 GeV pion showers contained in the AHCAL. A fit to the weights, used to parametrize the dependence, is also shown.

\[ \omega_j(E) = p_1(E) e^{p_2(E) + j} + p_3(E), \quad (4.2) \]

with \( j \) the index of the energy density bin running from 1 to 10. This has the additional advantage of simplifying the minimization procedure by reducing the number of parameters in the process. The behavior of the minimization procedure is also improved since the function enforces a smooth behavior of the weights with local density, thus eliminating fluctuations. The three parameters \( p_1, p_2, p_3 \) in general are energy dependent. The three parameters are also highly correlated in the fit. The following shows the correlation matrix for a 20 GeV run.

\[
\begin{pmatrix}
1.000 & 0.928 & -0.975 \\
0.928 & 1.000 & -0.984 \\
-0.975 & -0.984 & 1.000
\end{pmatrix}
\quad (4.3)
\]

The parameters \( p_1, p_2, p_3 \) were determined for each run in the data set using a minimization procedure analogous to the one described in Equation \[4.1\] using the function given in Equation \[4.2\] instead of the individual weights. To avoid correlations between the weight determination and the subsequent analysis, the energy dependent weights were determined from the first half of the events in all runs using the above method. The parametrized weights were then applied to the second half of the runs.

To improve the stability of the determination of the energy dependence of the three parameters, they were determined in an iterative procedure, first with all three parameters unconstrained in the minimization procedure. After this first iteration the energy dependence of parameter \( p_3 \) was parametrized with the following function

\[ p_3(E) = a_1 (1 - e^{a_2 E}). \quad (4.4) \]
Figure 4. Energy dependence of the three parameters used to describe the local shower density dependent weights together with phenomenological descriptions that allows smooth interpolation between the data points. When the energy dependent weighting is used in the data reconstruction, the parameters for the weighting function are chosen according to these descriptions based on the energy reconstructed with one single weight.

In the second iteration, this parameter was constrained to this functional form, and parameter $p_2$ was determined from a linear fit. In the last iteration, parameters $p_2$ and $p_3$ were constrained, and only parameter $a$ was left free in the minimization procedure. Its energy dependence was then parametrized by the function

$$p_1(E) = c_1 + E c_2 + c_3 e^{c_4 E},$$  \hspace{1cm} (4.5)

with the parameters determined from a fit. The three parameters together with these fits are illustrated in Figure 4. Error bars are not given for the parameters, since the determination of errors in the minimization procedure is not yet finalized. For this analysis, the three weight parameters are chosen according to these phenomenological descriptions, taking the reconstructed energy of the single weight method as input energy. With this method, the reconstructed energy and the corresponding energy resolution is calculated for each run in the data set.

4.3 Results

The energy of showers contained in HCAL has been reconstructed with different methods, as described in Section 4.2. The average reconstructed energy and the energy resolution for each run was determined from a two-step gaussian fit of histograms of the event-by-event distribution of the reconstructed energy. First, a Gaussian was fitted over the full range of the histogram. Then, a second Gaussian was fitted only in the range of $\pm 1.5 \sigma$ of the first fit. The mean and the $\sigma$ of this second fit were used as the mean reconstructed energy and as the energy resolution, respectively.

Figure 5 shows the reconstructed energy as a function of beam energy, both when using one single, constant weight factor, and when using the energy dependent, parametrized weights. It is apparent that in both cases the detector response is quite linear. The lower panel in the same figure shows the relative deviation of the reconstructed energy from the beam energy for the case of the reconstruction with a single conversion factor and for the energy dependent weights. It is
apparent that the parametrized energy dependent weighting significantly improves the linearity of the detector response, and brings all measurement within 5% of the beam energy. The spread for different runs at the same energy is caused at least partially by the missing temperature correction, and will be significantly improved once a cell-by-cell temperature correction for the data becomes available.

The energy resolution obtained with the different methods is shown in Figure 6 left. The black circles correspond to the calculation using one constant coefficient. The calculation with the individual weights is seen in the blue open circles. Note that in this case, the beam energy is used as a constraint in the minimization procedure. This method is not applicable in a real experimental situation, where the particle energy is not known a priori. As such, these points represent the energy resolution that is in principle reachable using the simple weighting method described here. The red points depict the realistic case, using energy dependent weights with a parametrized energy dependence. These weights were selected from the parametrizations using the energy reconstructed using the constant coefficient method. In this case, the parametrized weights reach essentially the same resolution as the method using the beam energy constraint. Fits to the data points are also shown. In the fit, the noise term was omitted since the noise rejection method reduced noise fluctuations in the AHCAL to zero within errors. The fit results show a significant

Figure 5. Reconstructed energy as a function of beam energy, both for the single weight method and the energy dependent parametrized weights. The lower panel shows the relative deviation of the reconstructed energy from the beam energy for the energy dependent reconstruction, both for the single weight method and the energy dependent weights.
Figure 6. Left: Energy resolution for showers contained in the HCAL for different reconstruction methods. The black points correspond to the single weight method. The open blue circles are calculated with individual weights, using a beam energy constraint. The red circles are found with the parametrized energy dependent weights. As energy input the reconstructed energy with one single weight was used in that case. Right: Ratio of energy resolution obtained with density dependent weights to the resolution obtained with a single weight factor.

improvement of the stochastic term of the energy resolution when the energy density dependent weighting is used.

Figure 6 right shows the ratio of the energy resolution obtained with the two energy density dependent weighting methods and the resolution obtained with the single conversion factor. In both cases, a significant improvement of the energy resolution is observed. In the case of the realistic reconstruction using energy dependent weighting, an improvement by approximately 25% was achieved over most of the energy range, with 15% to 20% improvement at low energy, and no improvement at the highest energy. This is likely due to the requirement of full containment of the shower in the AHCAL, which forces a high electromagnetic fraction at high energies, reducing the effectiveness of the weighting method. In addition, the containment requirement reduces the available statistics, which also has a negative impact on the weight determination.

5. Hadronic Energy Resolution of the Complete CALICE Setup

5.1 Event Selection

To reject muons and empty events as well as electrons, a minimum of 50 hits above a threshold of 0.4 MIPs and a minimum total energy deposit of 100 MIPs was required in the HCAL. In the analysis, only hits with an energy above 0.5 MIP were used. No temperature correction was applied, leading to a noticeable spread in the reconstructed energy for different runs taken at different times with the same beam energy. The variations of the mean HCAL temperature over the used data set were about 2.5 °C, with the majority of runs taken at mean temperatures of around 27.5 °C.

5.2 Analysis Technique

5.2.1 Noise Rejection

A noise rejection for all detectors was applied. For the AHCAL, the same method as discussed
in Section 4.2.1 was used. This technique was expanded to all three detectors. For each detector channel, a three dimensional energy density, taking the energy deposits of all neighboring cells in three dimensions into account, was calculated. For the ECAL and the AHCAL, this energy density was given by the sum of the energy in the cell under study as well as all the energy in all neighboring cells in the same layer and in the layers before and after the hit. In the case of the TCMT, were the active layers consist of alternately oriented scintillator strips, the energy deposit in the neighboring strips in the same layer and the total energy deposit in the layers before and after the cell under study are used. Noise hits have a low value for this 3D energy density, and can thus be distinguished from other energy deposits. The cut values were determined using pedestal events, were no beam particle was entering the detector. In addition to noise hits also here isolated neutrons are rejected.

For pedestal events, the noise rejection procedure reduced the mean energy in pedestal events from typically 650 MeV to 200 MeV. While the ECAL contributed very little to the noise also before the rejection algorithm, the contribution of the HCAL was reduced from 200 MeV to 25 MeV. The TCMT was the biggest contributer both before and after the noise rejection, partly due to the high weight factor of the last 8 layers. Before the rejection algorithm, the TCMT contribution was approximately 440 MeV, with 180 MeV remaining after the rejection. This noise rejection leads to a slight increase of the energy resolution in the case of single weight reconstruction. It was also observed that the linearity of the detector response with single weights was slightly worse when the noise and isolated neutrons are removed. The inclusion of noise adds a constant energy offset, independent of beam energy, and might thus also help to reduce the effect of an increased detector response for higher energy showers. Still, the noise rejection was used for all three energy reconstruction methods.

5.2.2 Energy Reconstruction with Single Weights

Since the complete CALICE setup consists of three separate detectors, ECAL, HCAL and TCMT, at least three conversion factors are necessary to convert the measured signals to a reconstructed energy. The detector signals are given in units of MIPs after the application the detector calibration. For the conversion from the MIP to the total energy scale, one factor per detector is used in the case of energy reconstruction with single weights, described in this subsection. For the ECAL and the TCMT, additional inter-calibration factors are needed to account for the different longitudinal samplings in different regions of these detectors. Thus in total 6 calibration factors are determined to describe the conversion from MIPs to Energy, three in the ECAL, one in the HCAL and two in the TCMT. These six factors $\alpha_i$ were determined from the data by minimizing the energy resolution for a given beam energy, analogous to the method used in [3]:

$$\chi^2 = (E_{\text{beam}} - \sum_{i=1}^{6} \alpha_i E_i)^2$$  \hspace{1cm} (5.1)

where $E_i$ is the total energy sum in MIPs in the sub-detector region $i$ (ECAL $i = 1, 2, 3$; HCAL $i = 4$; TCMT $i = 5, 6$). This procedure was performed for all runs in the data set. From this set of factors, an energy-independent mean inter-calibration factor was determined for each detector section. Table 1 shows these factors for all detector sections.
<table>
<thead>
<tr>
<th>Detector section</th>
<th>conversion factor [GeV/MIP]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECAL 1</td>
<td>0.0081 ± 0.0011</td>
</tr>
<tr>
<td>ECAL 2</td>
<td>0.0092 ± 0.0005</td>
</tr>
<tr>
<td>ECAL 3</td>
<td>0.0134 ± 0.0004</td>
</tr>
<tr>
<td>HCAL</td>
<td>0.0289 ± 0.0003</td>
</tr>
<tr>
<td>TCMT 1</td>
<td>0.0335 ± 0.0027</td>
</tr>
<tr>
<td>TCMT 2</td>
<td>0.1524 ± 0.0142</td>
</tr>
</tbody>
</table>

**Table 1.** Single weight factors from the MIP to the absolute energy scale for all six detector regions in the complete CALICE setup.

To facilitate the subsequent analysis, in particular the determination of conversion factors dependent on the local shower density discussed in Section 5.2.3, the conversion factors for the ECAL and for the TCMT were converted to one overall detector weight each and three inter-calibration factors for the case of the ECAL, two for the TCMT. In the subsequent analysis, energy deposits in the respective sections of the detectors were weighted with this factor in the overall energy sum of the detector. For the ECAL, the inter-calibration factors were thus 1 : 1.124 : 1.629, for the TCMT they were 1 : 4.55. Note that in particular in the case of the ECAL these factors are quite different from the ratio of the absorber thicknesses, showing that the intercalibration factors that yield the best energy resolution are not only given by the different sampling fractions, but are in addition influenced by correlations within the hadronic showers. The same behavior was observed in the first combined analysis discussed in [3].

Using the same minimization procedure as for the 6 detector section factors, three weight factors, one for each detector, were determined with the application of the inter-calibration factors in the ECAL and in the TCMT. These factors are given in Table 2 and are used in for the energy reconstruction based on single weights.

<table>
<thead>
<tr>
<th>Detector</th>
<th>conversion factor [GeV/MIP]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ECAL</td>
<td>0.00827</td>
</tr>
<tr>
<td>HCAL</td>
<td>0.0293</td>
</tr>
<tr>
<td>TCMT</td>
<td>0.0337</td>
</tr>
</tbody>
</table>

**Table 2.** Conversion factors from the MIP to the absolute energy scale for each detector in the complete CALICE setup, when using the inter-calibration factors for the individual sections in the ECAL and in the TCMT.

Using these conversion factors, the total corrected energy was calculated for each event according to

$$E_{\text{rec}} = (E_{\text{ECAL},1} + 1.124 E_{\text{ECAL},2} + 1.629 E_{\text{ECAL},3}) \times w_{\text{ECAL}}$$

$$+ E_{\text{HCAL}} \times w_{\text{HCAL}} + (E_{\text{TCMT},1} + 4.55 E_{\text{TCMT},2}) \times w_{\text{TCMT}}$$

(5.2)

where $E_{\text{ECAL},i}$ and $E_{\text{TCMT},i}$ are the total energy deposits in the individual sections of the ECAL
and the TCMT, $E_{HCAL}$ is the total energy deposit in the HCAL, and $w_{ECAL}$, $w_{HCAL}$ and $w_{TCMT}$ are the detector weights as given in Table 2.

For each analyzed run, the reconstructed energy was filled into a histogram. The mean reconstructed energy and the energy resolution was determined from a two-step Gaussian fit to these histograms, as discussed in Section 4.3. Figure 7 shows a selection of these energy histograms and the corresponding final fits. The resulting detector response and the energy resolution are further discussed in Section 5.3.

5.2.3 Energy Reconstruction with Energy Density Dependent Weights

In order to study the effect of local energy density dependent signal weighting, the data of each detector (ECAL, HCAL, TCMT) is binned according to the local energy density for each event. For the ECAL and the TCMT, the energy density is taken as the signal in MIPs per readout cell. Since the HCAL uses three different cell sizes, the HCAL energy density is defined as the signal per cell in MIPs, normalized to the volume of a $3 \times 3$ cm$^2$ cell. Thus, for the small cells, the definition of the energy density is the same as for the ECAL and the TCMT, while for the larger cells the cell signal is divided by 4 ($6 \times 6$ cm$^2$ cells) or 16 ($12 \times 12$ cm$^2$ cells). The density for each detector was subdivided into arbitrarily chosen bins. Those bins were used for the density dependent weighting procedure described below. Figure 8 shows the distribution of the local energy density in all three detectors for 25 GeV hadrons together with the subdivision into density bins.

The philosophy behind the weighting with energy density dependent weights is that electromagnetic sub-showers typically have higher energy densities than hadronic sub-showers, so a (partial) compensation of the detector response can be achieved by weighting energy deposits with high local density less than those with lower density. This feature was incorporated in the anal-
ysis by expanding the single weights (one per detector), discussed in Section 5.2.2, into separate weights for each detector and each energy density bin. The different sections in the ECAL and in the TCMT were again incorporated using the inter-calibration factors to weight energy deposits in different sections differently. In the calculation of the local energy density, these inter-calibration factors were not used, thus basing the local density measure on the particle density in a given read-out cell. 6 energy density bins were used in the ECAL and in the TCMT, and 10 bins were used in the HCAL, resulting in a total of 22 weights. Since most of the energy gets absorbed in the HCAL, a more precise determination of the weights is possible in this detector, motivating the use of more weights than in the other detectors.

These weights were determined using the same procedure as described previously, by minimizing the energy resolution for a given beam energy:

$$\chi^2 = (E_{\text{beam}} - \sum_{i=1}^{22} \alpha_i E_i)^2$$

where $E_i$ is the total energy sum in MIPs in energy density bin $i$, where $i$ spans across all three detectors: ECAL $i = 1$ to 6; HCAL $i = 7$ to 16; TCMT $i = 17$ to 22. With this method, an optimal set of weights was determined for each run in the data set.

With those weights, a significant improvement of the energy resolution over the resolution obtained with one single weight per detector was achieved. However, it was found that the optimal weights change strongly with energy, and also fluctuate significantly from run to run. These fluctuations are likely due to large uncertainties in the minimization procedure. In order to get a good linearity of the detector response together with an improved energy resolution, an energy dependent parametrization of the weights was required. This then allowed to use the energy density dependent weighting procedure without the knowledge of the beam energy by selecting the weights according to the parametrization using the energy determined with the single weights procedure as input.

To facilitate the description of the energy dependence by reducing the number of free parameters in the weight determination, a phenomenological parametrization of the density dependent...
weights in each detector was used. The same functional form as for the HCAL only case, given in Equation 4.4 was used, generalized to three detectors:

\[ w_{j,d}(E) = p_{1,d}(E) e^{(p_{2,d}(E) \cdot j)} + p_{3,d}(E) \]  

(5.4)

where \( w_j \) is the weight for the \( j^{th} \) density bin in a given detector \( d \), counting from 1 to 6 in the ECAL and TCMT and from 1 to 10 in the HCAL. \( p_1, p_2 \) and \( p_3 \) are the free parameters of the function. This reduces the number of parameters to describe the density dependent weighting from 22 to 9. Similar performance in terms of energy resolution was obtained when determining these parameters with a minimization procedure for each run using the beam energy constraint, compared to the procedure with 22 independent weights.

The determination of all 9 parameters still resulted in significant fluctuations, so the energy dependence of these parameters was determined in an iterative procedure. First, \( p_3 \) for all detectors was fitted with the energy dependent function

\[ p_3(E) = a_1 (1 - e^{a_2 E}) . \]  

(5.5)

The determined parameters were then used in a subsequent minimization step to determine \( p_1 \) and \( p_2 \). In the following step, \( p_2 \) for all detectors was fitted with a linear function as a function of energy, leaving only \( p_1 \) unconstrained for the final iterative step. The result of this step is illustrated in Figure 9. The energy dependence of \( p_1 \) is parametrized with a phenomenological function of the form

\[ p_1(E) = c_1 + Ec_2 + c_3 e^{c_4 E} . \]  

(5.6)

This function was chosen to give reasonable descriptions of the energy evolution of the first parameter in the weight function. It uses four parameters, denoted by \( c_i \), where \( i \) represents the parameter index. Improvements of the overall performance of this reconstruction method might be achieved by finding better parametrizations of all three parameters, potentially using different functional
forms for each detector. The fits performed to the first parameter are also shown in Figure 9. Error bars on the individual points are not shown since the error estimates from the minimization procedure are not reliable at present.

To avoid correlations between the weight determination and the subsequent analysis, the energy dependent weights are determined from the first half of the events in all runs using the above method. The parametrized weights are then applied to the second half of the runs.

Figure 10. Distribution of reconstructed energies for the parametrized energy density dependent weight technique for a selection of runs from the total data set (one run per energy). The fits used to extract the mean reconstructed energy and the energy resolution are also shown.

The energy in the complete CALICE setup was reconstructed using the energy dependent parametrization of weights depending on the local shower density by choosing the parameters for Equation 5.4 based on the energy estimated from the single detector weights as described in Section 5.2.2. The first parameter was taken from Equation 5.6. The other two parameters were taken from the linear parametrization for parameter 2, and from Equation 5.5, as discussed above.

The mean reconstructed energy and the corresponding energy resolution for each run in the data set was determined again by gaussian fits to the distribution of reconstructed energies within $\pm 1.5 \sigma$ of the peak. Figure 10 shows a selection of these distributions together with the fits. The comparison to the same distributions obtained with single weights per detector, shown in Figure 9, gives a first indication of the improvement in energy resolution that is obtained with the local shower density dependent weights, based on the energy dependent parametrization discussed here.

### 5.3 Results

Preliminary results for the reconstructed energy and the energy resolution for the complete CALICE setup have been obtained with three different analysis techniques:

- Energy independent single weights (one factor per detector), described in Section 5.2.2.
Figure 11. Reconstructed energy of the complete CALICE setup for hadrons at various energies as a function of beam energy. The lower panel shows the relative deviation of the reconstructed energy from the true beam energy.

- Energy dependent parametrization of weights depending on the local shower density, described in Section 5.2.3. The data set used in the analysis was statistically independent from the data set used to determine the weights.

- Individual weights depending on the local shower density, determined with a beam energy constraint for each data point, described in Section 5.2.3.

Figure 11 shows the reconstructed energy for hadrons over a wide range of beam energies using both single detector weights and the energy dependent parametrization. Also shown is the relative deviation from the beam energy, \((E_{\text{reconstructed}} - E_{\text{beam}})/E_{\text{beam}}\) in the lower panel. Both methods of reconstruction yield a very satisfactory linearity of the detector response, in particular considering that no temperature corrections were applied. The application of a cell-by-cell correction of temperature effects in the HCAL is expected to reduce the spread between different runs considerably. In many cases for the single weights and in all cases for the energy dependent parametrization the deviation from the beam energy is less than 5%. This good linearity is particularly surprising for the case of single weights, since an increasing detector response with increasing electromagnetic fraction of the showers, and thus with increasing energy, is expected. The saturation of the SiPMs in the HCAL for high signals due to the finite number of pixels in the devices, and the incomplete correction for this effect might be partially responsible for this behavior. It reduces the response
for cells with high energy density, and thus works in the same direction as the weighting methods discussed here.

Figure 12. Energy resolution of the complete CALICE setup for hadrons at various energies as a function of beam energy.

Figure 12 shows the energy resolution for all three analysis techniques, together with fits to the data points. Not surprisingly the case with density dependent weights determined with a beam energy constraint for each data points yields the best resolution. This case illustrates the best possible energy resolution achievable with the simple weighting technique discussed in this note, but is not applicable in a realistic detector, since the correct particle energy is not known in this case. The use of single detector weights shows the energy resolution reachable with the calorimeter system without making use of the additional information provided by the high granularity of the detectors. Compared to this, the use of an energy dependent parametrization of the weighting according to the local shower density yields a significant improvement of the energy resolution. Note that the energy reconstructed with single detector weights and not the beam energy is used to determine the weights according to the parametrization discussed in Section 5.2.3. The fit results show that the energy dependent weighting improves the stochastic term of the energy resolution significantly.

The improvement of the energy resolution compared to the resolution obtained with single detector weights, given by the ratio of the resolutions, is shown in Figure 13. The use of the energy dependent parametrization improves the resolution by typically 18%, with less improvement of only 10% to 15% below energies of 15 GeV. The comparison to the resolution obtained with the weights determined for each run separately using a beam energy constraint suggests that an improvement of up to 23% might be obtained with the simple weighting procedure described here if an optimal parametrization of the weights is found. In practice this will be also limited by the imprecise knowledge of the shower energy that goes into the selection of the weights according to the parametrization. Both for the AHCAL only and for the complete setup comparable resolutions
6. Summary

An initial study of the hadronic energy resolution and the linearity of the detector response for hadronic showers contained in the analog HCAL and for the complete CALICE setup has been performed, using data taken in 2007 at CERN. Simple weighting methods based on the local shower density, given by the energy in a particular detector cell, have been used to improve the energy reconstruction and the resolution. Both for the showers contained in the HCAL and for the complete CALICE setup, significant improvement of the energy resolution was observed when using a weighting algorithm based on a simple functional form for the density dependent weights including a parametrization of their energy dependence. In the case of the HCAL only analysis, an improvement of the energy resolution by up to 25% compared to the use of a single, energy independent weight factor, was observed. In the case of the complete CALICE setup, the improvement was on the order of 18%. In both cases, a stochastic term of the order of $50\% / \sqrt{E}$ was reached with the weighting procedure. The application of the weights also improved the linearity of the detector response in both cases. The comparison to the resolution obtained with weights optimized separately for each beam energy using a beam energy constraint showed that some additional improvement can likely be achieved for the complete CALICE setup, in particular at low energy. Further improvement is expected once a cell-by-cell temperature correction for the data becomes available.
References


Appendix

A. List of Runs

The table shows the runs used in the analysis presented here, together with the beam energies for each run. All runs were taken in 2007 at CERN, with the complete CALICE setup and with the HCAL not rotated. All runs are hadron runs, no particle identification is used (although available for some of the runs). The data was reconstructed locally in Munich using CALICE software version 04-06-03 (calice_userlib v04-10, calice_reco 04-06, marlin v00-09-10). This version of the reconstruction uses a preliminary saturation correction based on the saturation curves measured at ITEP, and uses a global scaling factor of 0.8 to correct for the not perfect coupling of the wavelength shifting fibers to the SiPM.

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Table 3. Runs used in the Analysis.
B. Additional Figures

This appendix contains additional figures which can be useful for presentations. These figures show the resolution as well as the ratio of the resolutions with and without weighting without the data points for the method using the beam energy constraint.

B.1 HCAL Only Analysis

![Figure 14](attachment:image1)

**Figure 14.** *Left:* Energy resolution for showers contained in the HCAL for two different reconstruction methods. The black points correspond to the single weight method. The red data points show the resolution obtained with the parametrized energy dependent weights. *Right:* Ratio of energy resolution obtained with density dependent weights to the resolution obtained with a single weight factor. This figure is a variation of Figure 6.

B.2 Complete CALICE Setup

![Figure 15](attachment:image2)

**Figure 15.** *Left:* Energy resolution of the complete CALICE setup for hadrons at various energies as a function of beam energy, showing the single weights method and the method using parametrized energy dependent weights. *Right:* Ratio of the energy resolution using the parametrized weights compared to the case with single weights per detector, illustrating the improvement achieved with weighting procedures based on local energy density. This figure shows variations of Figures 12 and 13.