Tracking within Hadronic Showers in the SDHCAL prototype using Hough Transform Technique

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ABSTRACT: Hadronic showers such as those collected in the Semi-Digital Hadronic CALorimeter (SDHCAL) include tracks produced by charged particles such as Minimum Ionising Particles (MIP). These tracks could be used as a tool to probe the behaviour of the SDHCAL active layers in situ. To extract these tracks the Hough Transform method is used in this work. The results of this study indicate clearly that the Hough Transform is highly efficient when it is applied after elimination of the hits belonging to the dense parts of the hadronic showers. The method is also applied to events simulated in the SDHCAL with different hadronic shower models and the results are compared to those obtained from data.
1. Introduction

Hadronic showers produced by the interaction of hadrons with the absorber part of a sampling calorimeter like the SDHCAL prototype [1] contain often several tracks associated to charged particles. Some of these particles could cross many active layers before being stopped and others could even leave the calorimeter. An imaging calorimeter could build such tracks and use them to monitor its own behaviour and more precisely that of its active layers in the presence of the hadronic shower [2]. This allows one to understand better the response of the calorimeter and to use the collected information to estimate optimally the hadronic energy. The capability of the calorimeter to construct such tracks depends on its granularity. The higher the granularity the lower the confusion between such tracks and the remaining parts of the hadronic shower. However, even with a highly-granular calorimeter it is difficult to separate the tracks from the highly dense environment which is present in hadronic showers produced by high-energy hadrons. To achieve such separation one could use the Hough Transform (HT) method [3] that was developed many years ago to find tracks in a noisy environment. This method has already been proposed to extract MIP from the electromagnetic shower in the Si-W CALICE electromagnetic calorimeter prototype. The CALICE note
presents a detailed study of this application. In this paper we present the results obtained by applying the HT method to the hadronic showers collected during the exposure of the SDHCAL prototype to hadron beams at CERN in 2012. The second section introduces the HT method in general. In section three we explain how the method is applied in the case of dense environment such that encountered in hadronic showers. Section four presents the results obtained with this method when applied to hadronic showers of different energies. In section five the method is used to estimate the number of tracks and their length in simulated events using different models and a comparison among these models based on these new variables is presented. The same method is applied to electromagnetic showers produced during the exposure of the SDHCAL prototype to electron beams of different energies. Results are compared with those of hadronic showers. In section six, the use of the HT tracks as a tool to monitor the efficiency and pad multiplicity of the active layers of the hadronic calorimeter is presented and other applications related to Particle Flow Algorithms (PFA) are mentioned.

2. Hough Transform method

Hough Transform method is a simple and robust one that allows to distinguish aligned points from others. It permits also to find the parameters associated to the tracks built from the aligned points. The scope of this method is larger than the one we will present here but all the variants of this method are based on the same principle. To search for points located on a straight-line in a plane (say \((z, x)\)) the cartesian coordinates of each point present in the plane are replaced by a curve (hence infinite number of points) in the associated polar plane \((\theta, \rho)\):

\[
\rho = z \cos \theta + x \sin \theta
\]

In this transformation the aligned points have their curves intersecting in one point of the polar plane \((\theta^0, \rho^0)\) and form a node. The node’s coordinates in the polar plane determine the angle of the straight-line with respect to the ordinate axis of the \((z, x)\) plane and the distance of the straight-line to its origin point as can be seen in Figure 1. Curves associated to the other points intersect also with those of the aligned ones. In low points density these intersections scarcely coincide with \((\theta^0, \rho^0)\). Therefore to find aligned points one should look for nodes in the \((\theta, \rho)\) plane. The number of intersecting curves in one node is an essential parameter to estimate the number of aligned points. In dense environment the same method could be applied. However, one should take into account here the possibility that the points contributing to one node do not necessarily belong all to the same track.

The method described above could not be applied as such to the hits left by particles in a given detector. The spatial resolution of the hits position in the detector and more importantly the Multiple Coulomb Scattering of the associated charged particles require the method to be adapted accordingly. This can be achieved by discretising the \((\theta, \rho)\) plane and transforming it into a histogram. The bins of this histogram are incremented each time they are crossed by a curve associated to one point in the \((z, x)\) plane. The size of histogram bins reflects the expected resolution associated to hits belonging to one particle and could be optimized to help finding not only straight-line tracks but also slightly curved ones. In this way the nodes of intersecting curves are replaced by bins
Figure 1: Illustration of the Hough Transform method. Each point on the left figure has its associated curve on the right one with the same colour. The curves associated to the points located on the straight line, intersect in the same point in the $(\theta, \rho)$ plane.

whose content is related to the number of aligned hits.

3. Use of Hough Transform in presence of the hadronic shower

SDHCAL is made of 48 active layers interleaved with 2 cm absorber plates made of stainless steel. Each active layer is made of $96 \times 96$ of $1 \text{ cm}^2$ pads. Two adjacent pads are separated by 0.408 mm. Hits left by charged particles in one active layer are represented by fired pads. A small number of pads could be fired by the passage of one charged particle in an active layer. The average value of these number is called the pad multiplicity.

Hadronic showers are generally characterized by a dense core (electromagnetic part) located in the centre and less dense part (the hadronic one) in the periphery. Therefore, to use the Hough Transform to find tracks within hadronic showers one should avoid using hits located in the dense core since one can build artificially many tracks from the numerous hits of the core. This can be achieved by keeping only hits that have few neighbours. To avoid unnecessary time consuming, the Hough Transform method is applied not to the hits themselves but rather to clusters of hits resulting from aggregating topologically neighbouring hits in one active layer ($(x, y)$ plane). Those clusters are built recursively. The first cluster is built starting from one randomly chosen hit of the $(x, y)$ plane. Adjacent hits to this first hit are looked for and are added to the cluster. Hits adjacent to any of the cluster hits are again looked for and added. The procedure is applied until no new adjacent hit is found. The hits belonging to this cluster are tagged and withdrawn from the hits list. The same procedure is applied to a new hit randomly chosen among the remaining hits of the plane. The procedure is repeated until all the hits are gathered into clusters. The coordinates of a resulting
cluster are that of the geometrical barycentre of its hits. In our case, clusters are rejected if they have more than 2 neighbouring clusters in an area of $10 \times 10 \text{ cm}^2$ around or if one cluster of more than 5 hits is found in this area. The selected clusters are used to fill a 2D histogram containing 100 equal bins in $\theta_1$ covering the angular range $[-\frac{\pi}{2}, \frac{\pi}{2}]$ and 150 equal bins of 1 cm for $\rho_1$. The 150 cm value corresponds to the maximum value of $\rho$ a track may have in the $100 \times 130 \text{ cm}^2$ of the $(z,x)$ detector plane. For each cluster determined by its $x$ and $z$ positions and for each $\theta_i$, $i = 0 \cdots 99$ the integer value of the associated $\rho^i$ is calculated. This allows to add one entry to the 2D bin of $(\theta^i_1, \rho^i_1)$. After filling the histogram, bins with more than 6 entries are chosen. The choice of 6 is a compromise between finding tracks long enough to perform elementary efficiency studies and at the same time selecting tracks associated to low energy particles suffering from multiple scattering effect.

To eliminate tracks made of topologically uncorrelated clusters, each of those associated to one selected bin is compared to the others. The cluster is kept if at least two other clusters are found in a cube of $18 \times 18 \times 18 \text{ cm}^3$ centred on this cluster.

To eliminate the scenario of accidentally aligned clusters in the 2D $(z,x)$ plane and not in the 3D $(z,x,y)$ space, the $(z,y)$ coordinates of the clusters belonging to one selected bin are used to fill a second histogram $(\theta_2, \rho_2)$ with the same binning. If any of the $(\theta_2^i, \rho_2^i)$ bin is found with more than 6 entries then only the clusters which contribute to this bin are kept. The clusters found in this way are then used to build a track whose parameters are determined from the four values : $\theta_1^i, \rho_1^i, \theta_2^i, \rho_2^i$.

4. Hough-Transform tracks within hadronic showers in the SDHCAL

![Image](image_url)

Figure 2: Left : 50 GeV muons in the SDHCAL. Red points are the identified hits belonging to a track while black points are not. Right : Associated $(\theta_1, \rho_1)$ histogram which corresponds to the transformation of the $(z,x)$ plane.
As a first step to validate the method described in the previous section, we apply it to cosmic and muon events collected in SDHCAL prototype during its exposure to particle beams at CERN-SPS in 2012. Most of the hits belonging to muon events were found to be well selected while noisy hits outside the particle path are not as can be shown in Figure 2.

The Hough Transform method was then applied to events containing hadronic showers produced by pion beams. The selection of these events is based on criteria presented in a previous CALICE note [1]. Contaminations by electrons, beam muons as well as cosmics that may affect this study are analysed and criteria to eliminate them efficiently are explained in that note. The noise contribution was also studied and found to be of the order of one hit per event which has negligible effect on the present study even for hadronic shower of 5 GeV.

The selected hadronic showers cover a large range of energies from 5 to 80 GeV. Many of the tracks of the hadronic showers at high energy are associated to low energy particles which suffer from Multiple Coulomb Scattering deviation but still able to cross few active layers. To select such tracks either we select Hough Transform histogram bins with low number of clusters or we increase the θ bin size of this histogram. The later option is technically simpler to apply and therefore it was chosen.

For efficiency study one would like to have long tracks with a good $\chi^2$. These tracks are obtained by selecting bins with large number of clusters. Figure 3 shows two event displays of hadronic showers at 30 and 80 GeV with the Hough Transform selected hits. An important aspect of this method is the easy way to single out the tracks associated to the incoming charged particle in the calorimeter as can be seen in Figure 4. This is an interesting feature that could be used in the particle flow approach.

Figure 3: 80 GeV (left) and 30 GeV (right) hadronic showers with hits belonging to tracks in red.

5. Tracks in hadronic shower models

The number of tracks produced in showers collected in SDHCAL prototype can be used as a tool
Figure 4: 80 GeV pion in the SDHCAL. The track (in red) before the shower starts is well identified and could be useful for PFA.

to compare the different hadronic shower models. The number of tracks is related to the number of charged particles (pions, kaons and protons) produced in the hadronic shower with enough energy to cross few absorber layers. Events with different energies produced by pion interaction in SDHCAL prototype were simulated using three hadronic shower models within the GEANT4 framework\(^1\). The three models are the \textit{FTFP\_BERT\_HP}, the \textit{QGSP\_BERT\_HP} and the \textit{LHEP} \([\text{6}]\). A digitizer to transform the energy deposed by the particles crossing the active layers into charges that induce signal in the neighbouring pads was used for the simulation. In the case of a single charged particle crossing one pad of the active layer the digitizer’s parameters were tuned to reproduce fairly the efficiency and the pad multiplicity observed with beam muons (see Figure [11] in Appendix \(\text{A}\)). The difference in efficiency of \(\% 3\) between data and simulation could be explained in part by the fact that the effect of dead channels in data was not introduced in the simulation for this work.

To account for the charge screening effect when more than one particle crosses the same pad, one additional parameter is used. The new parameter is a distance above which the charges of two adjacent particles are added and below which only one of the two charges is randomly considered. Although this is not the ideal way to deal with the charged screening effect, it should allow rather a better description of reality with respect to a simple addition of charges. The parameter was tuned so the total number of hits associated to hadronic showers observed in data are well reproduced by the simulation for the different studied energies. To achieve this the \textit{FTFP\_BERT\_HP} was used for the parameter optimization. However, when the same parameter was used in the two other models to simulate events, the agreement with data was found to be optimal as well (see Figure [12] in Appendix \(\text{A}\)). The use of the total number of hits observed in data to optimize the screening

\(^1\)The 9.6p01 version.
The effect parameter of the digitizer should in principle have no consequence on the study of the number and the length of the tracks produced in the different hadronic shower models with respect to that found in data since only single tracks are concerned here. However, it allows to account better for the tracks environment in the simulation which constitute a kind of background one should correctly simulate for the HT study.

Finally, the same digitizer with the same set of parameters was used to simulate events with the three models used in this analysis for all the studied energy.

The tracks obtained using the HT in simulated events with the three models are compared to each other and to data for different energies. Figure 5 shows the distributions of the total number of reconstructed tracks within 10, 40 and 70 GeV hadronic showers. The track length could be an interesting variable to compare simulation models with data. It is defined as the distance between the most upstream cluster position with respect to the beam direction and the most downstream one of the clusters belonging to a given HT selected track. Figure 5 shows the track length distribution for different energies of data and simulated events. In Appendix B and C further distributions of these variables are shown. Figure 7 shows the mean number of reconstructed tracks and their length as a function of the incoming particle energy. Even though the systematics uncertainties related to the difference of efficiency of single tracks between data and simulation which is at the level of 3% is yet to be included in this analysis, it appears that QGSP_BERT_HP model is closer to data than FTFP_BERT_HP one, specially for the number of reconstructed tracks. However, for both models the number of reconstructed tracks is higher than the number of those found in data and they are shorter. With significantly less reconstructed tracks, LHEP model differs from the two other models and from data. It features also shorter tracks with respect to the other models and to data especially at low energy.

Figure 5: Number of reconstructed tracks in hadronic shower for simulation and for data at 10, 40 and 70 GeV. The digitizer used in the simulation was tuned using hadrons data as explained in the text.

The same method was applied to rather pure electron events in the energy range from 5 to 70 GeV collected by using a special filter during the 2012 SDHCAL Test Beam as explained in CALICE note [1]. The absence of such tracks in the case of electrons as expected, compared with pions as can be shown in Figure 8 confirms the power of the Hough Transform method and the low probability that it introduces fake tracks in dense environment similar to the one prevailing in the hadronic shower core.
6. Use of Hough Transform tracks for PFA and calibration purposes

The tracks one can extract from hadronic showers play an important role to check the active layer behaviour in situ by studying the efficiency and multiplicity of the detector. For this the selected Hough Transform segments that exhibit a good $\chi^2$ behaviour are used. To study one layer, only clusters belonging to the other layers are kept and a new fit of the segment parameters in both $(z,x)$
Figure 8: Number of reconstructed tracks in showers produced by electron (red line) and pions (filled histogram) at 10, 40 and 70 GeV.

and \((z,y)\) projections is performed. The impact of the straight line to which the segment is associated in the studied layer is determined. The efficiency is then estimated by looking for clusters in a 2 cm radius around the impact. If at least one cluster is found, then the multiplicity is estimated by counting the number of hits associated to the closest cluster. Figure 9 shows the efficiency and the multiplicity per layer for a 40 GeV pion run. These results are generally consistent with what was observed in [1] where those two variables were estimated with beam muons and cosmics. The slight difference with the results obtained with the muon beam is probably related to the fact that the HT tracks are essentially concentrated in the SDHCAL centre. Study of local efficiency and multiplicity should allow to have a fair comparison in the future.

Figure 9: Efficiency and multiplicity per layer for a 40 GeV pion run.

The Hough Transform tracks can be very helpful in the PFA studies as well. They can be used to disentangle the close-by hadronic showers by connecting clusters produced by hadronic interaction of secondary charged particles to the main one as can be seen in Figure 10.
Another advantage one may have in extracting the segments is to use them for a better energy reconstruction. In the SDHCAL energy reconstruction method, each of the thresholds has a different weight [3]. Tracks of low energy that stop inside the calorimeter may have hits of second or third threshold. This may bias the energy estimation. Therefore giving the same weight for all the hits belonging to these tracks should improve on the energy reconstruction.

7. Conclusion

The Hough Transform is a simple and powerful method for finding tracks within a noisy environment. The technique to use this method to extract tracks in hadronic showers was applied to events produced by the exposure of the SDHCAL to hadron beams. The parameters of this technique have been detailed and allowed to have an efficient extraction. This method was also applied to simulated hadronic showers. Comparison with data allows to discriminate the different hadronic shower models used in the simulation. The advantages of using Hough Transform tracks to calibrate the hadronic calorimeter in situ are presented. In addition these tracks can be a useful tool in the PFA techniques. The extension of this technique to hadronic showers in the presence of magnetic field is being worked out. For high energy tracks whose trajectory is weakly affected by the magnetic field the same method could be used. For those of low energy, their trajectory is well characterized. The projection of these trajectories to the plane perpendicular to the magnetic field is a circle-like. Hough Transform method can be then used to find those circles in an appropriate way.

References

[1] The CALICE Collaboration, First results of the CALICE SDHCAL technological prototype, CALICE Analysis Note CAN-037, 30th November 2012

Figure 10: 50 GeV hadronic shower illustrating that connection between clusters could be done with the reconstructed tracks.


A. Digitizer control

Figure 11: Efficiency (left) and pad multiplicity (right) of the 48 layers as estimated from the simulation without including the effect due to dead channels in the data.
Figure 12: Total number of hits in hadronic showers as a function of the beam energy (a) and the relative deviation of the simulation models with respect to data (b). Black crosses correspond to data. Red squares correspond to \texttt{FTFP\_BERT\_HP} physics list, blue triangles to the \texttt{QGSP\_BERT\_HP} one and green triangles to \texttt{LHEP}. The digitizer used in the simulation was tuned using hadrons data as explained in the text.
**B. Number of reconstructed tracks**

Figure 13: Number of reconstructed tracks in hadronic shower using HT method for simulation and for data from 10 to 80 GeV. The digitizer used in the simulation was tuned using hadrons data as explained in the text.
Figure 14: Number of reconstructed tracks in showers produced by electron (red line) and pions (filled histogram) from 10 to 70 GeV.
C. Length of reconstructed tracks

Figure 15: Track length in hadronic shower for simulation and for data from 10 to 80 GeV. The digitizer used in the simulation was tuned using hadrons data as explained in the text.