Evaluation of maximum likelihood fits on GPU devices using CUDA

Felice Pantaleo
European Organization for Nuclear Research (CERN), Geneva, Switzerland
University of Pisa, Italy

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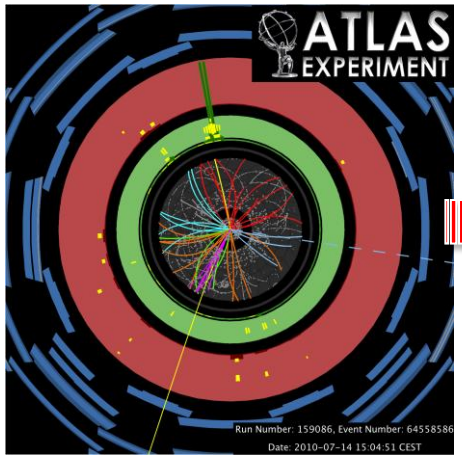
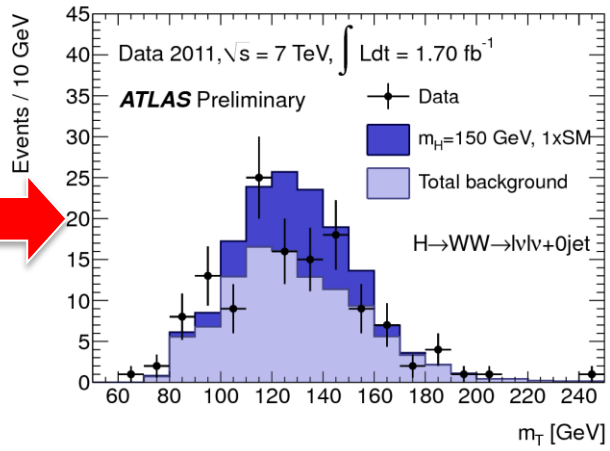
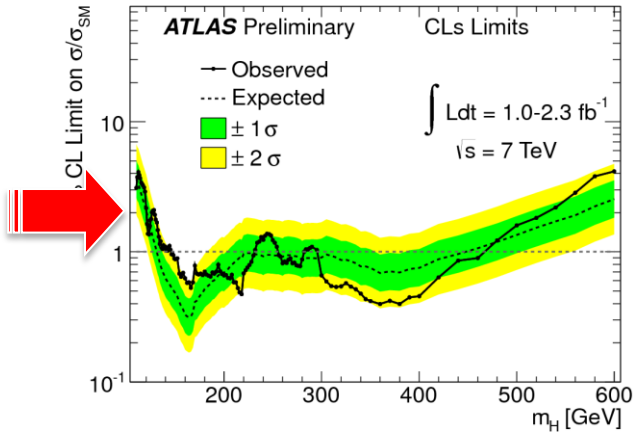
Data Analysis

- Huge quantity of data collected, but most of events are due to well-know physics processes
  - New physics effects expected in a tiny fraction of the total events: few tens
- Crucial to have a good discrimination between interesting (*signal*) events and the rest (*background*)
  - Data analysis techniques play a crucial role in this "fight"







Likelihood-based Techniques

- Data are a collection of independent events
  - an event consists of the measurement of a set of *observables* (energies, masses, spatial and angular variables...) recorded in a brief span of time by the physics detectors

- Introducing the concept of probability P (= Probability Density Function, PDF) for a given event to be signal or background, we can combine this information for all events in the *likelihood function*

$$\mathcal{L} = \prod_{i=1}^{N} \mathcal{P}(\hat{x}_i|\hat{\theta})$$

N number of events
$\hat{x}_i$ set of observables for the event i
$\hat{\theta}$ set of parameters

- Several data analysis techniques requires the evaluation of L to discriminate signal versus background events
- Finding the maximum of this function is equivalent to “what is the parameter estimation that makes the data set most probable for the prediction model?”

Algorithm Description

- Likelihood maximization is equivalent to minimize:

$$NLL = \sum_{j=1}^{s} n_j - \sum_{i=1}^{N} \left[\ln \sum_{j=1}^{s} \left(n_j \prod_{v=1}^{n} \mathcal{P}_j^v(x_i^v|\hat{\theta}_j) \right) \right]$$

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① Each P (Gaussian, Polynomial,…) is implemented with a corresponding class (basic PDF)

- Loop over the observables, produce a vector of results

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④ Reduction of all values

Parallelization Strategy

- All the calculations are done in double precision
- Data and results are organized in vectors
  - Input data read-only during the NLL evaluation
- Several loops to handle:
  - Evaluation and normalization of the basic PDFs, combinations, reduction
- Easy loop parallelism
  - Evaluation of functions over arrays of data
    - Balanced independent iterations (except the reduction)
  - Possible to exploit vectorization
    - Using Intel compiler for the auto-vectorization of the loops (using svml library by Intel)
- Reduction in parallel using block-wise algorithm, taking in account rounding problems

Implementations

I. OpenMP

- An OpenMP parallel region for each loop
  - Static partition of the iterations

II. CUDA

- Offload the loops to the GPU using CUDA kernels
  - A CUDA thread per each iteration
- Observables arrays are copied on the GPU once (synchronous)
- Results arrays are allocated on the GPU global memory once and they are deallocated at the end
- Reduction performed on the GPU, result (one double) copied to the host
- Using an heuristic approach to decide the block size:
  - 64 for kernels with transcendental functions, 128 otherwise
  - Occupancy ranges from 0.33 to 0.67, depending on the kernel

Test

$$n_a G_1^a(x) G_2^a(y) G_3^a(z) + n_b G_4^b(x) G_5^b(y) G_6^b(z) +$$
$$n_c A_1^c(x) P_1^c(y) P_2^c(z) + n_d P_3^d(x) P_4^d(y) A_2^d(z) +$$
$$n_e P_5^e(x) G_7^e(y) A_3^e(z)$$

Model from B. Aubert *et. al.*,
Phys. Rev. D80, 112002, 2009

21 PDFs in total, 3 observables, 5 types

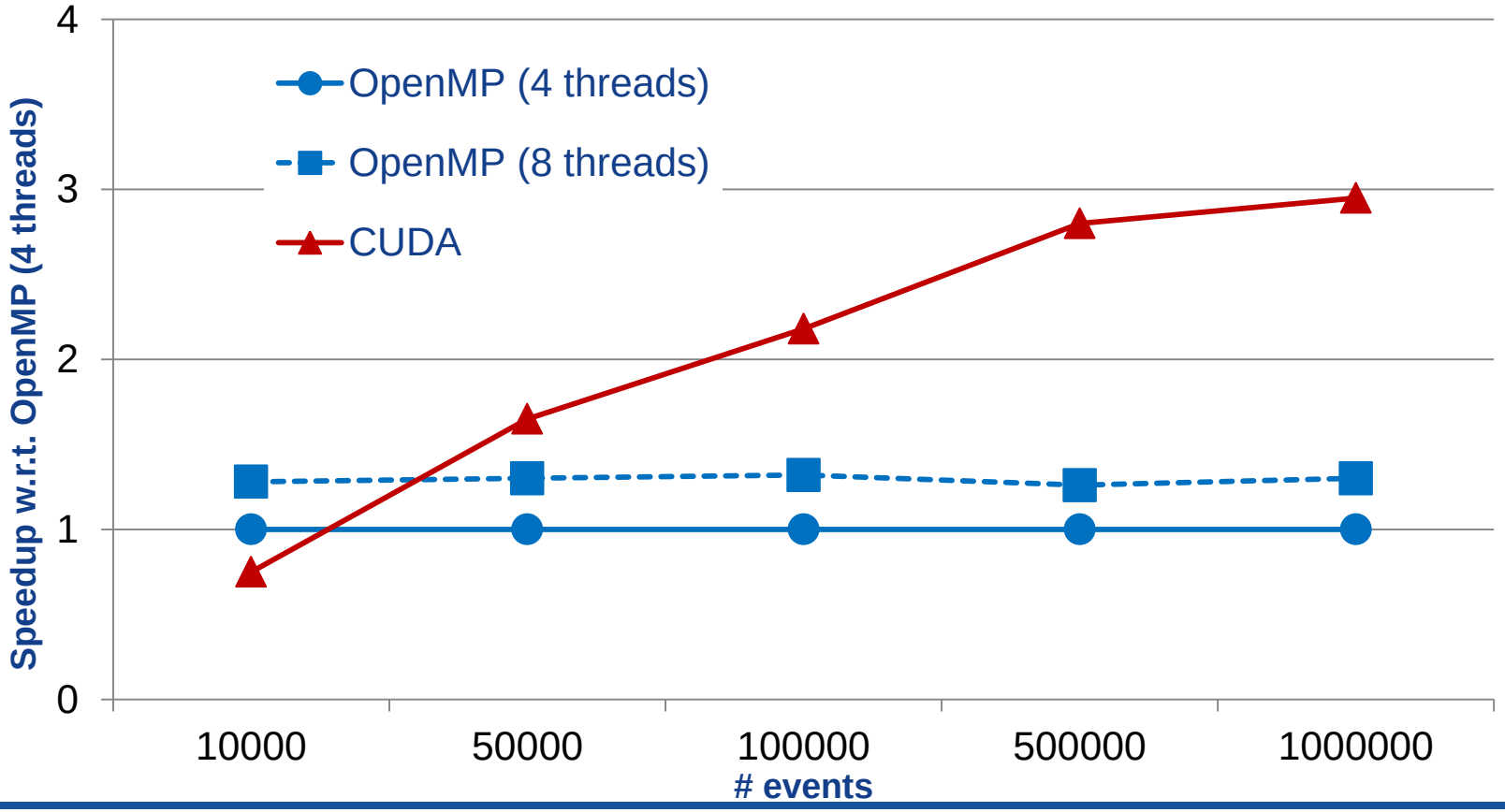
- G: Gaussian
- A: Argus function
- P: Polynomial

60% of the execution time is spent in exp's calculation

Note: all PDFs have analytical normalization integral, i.e. ~97% of the sequential portion can be parallelized

Results

- OpenMP CPU implementation on Intel 4-core “Nehalem”-based desktop system (Intel Core i7 965 @ 3.2GHz) compiled with Intel Composer XE 2011
  - Speed-up is 3.6x with 4 threads
  - Limited by sequential portion of the code
- CUDA GPU version implementation on NVIDIA GTX470, CUDA v4.0



Conclusion

- OpenMP CPU implementation
  - Good scalability and overall performance
- Successfully ported the algorithm using CUDA
  - Good performance on GPU cards
- More data will be collected by LHC experiments in the following years
  - GPU implementation will play an important role
- This is still a R&D project
  - Move to production code already started, which involves several groups in the experiments for testing
- Working on heterogeneous CPU+GPU (OpenMP + CUDA) and multiple GPUs systems.

Previous work references

- S. Jarp *et. al.*, *Parallelization of maximum likelihood fits with OpenMP and CUDA*, proceeding of "The International Conference on Computing in High Energy and Nuclear Physics", Taipei (Taiwan),18-22 October, 2010, to be published on Journal of Physics: Conference Series. CERN-IT-2011-009
- S. Jarp *et. al.*, *Evaluation of likelihood functions for data analysis on Graphics Processing Units*, ipdpsw, pp.1349-1358, 2011 IEEE International Symposium on Parallel and Distributed Processing Workshops and PhD Forum, 2011
- S. Jarp et. al., *Parallel Likelihood Function Evaluation on Heterogeneous Many-core Systems,* presented at International Conference on Parallel Computing, 30 August, 2011, Ghent (Belgium)

Q & A

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