The starting version of the present text contains the setup adopted in `arXiv:1606.02330` .

1 Setup for study of $p_\perp^Z$ and $p_\perp^W/p_\perp^Z$

- **Lagrangian constants**
For the numerical evaluation of the cross sections at the LHC ($\sqrt{s} = 8$ TeV) (shall we consider 13 TeV instead?) we choose the following set of Standard Model input parameters [1]:

$$\begin{aligned}
G_\mu = 1.1663787 \times 10^{-5}\ \text{GeV}^{-2}, &\quad \alpha = 1/137.035999074, \quad \alpha_s \equiv \alpha_s(M_Z^2) = 0.12018 \\
M_Z = 91.1876\ \text{GeV}, &\quad \Gamma_Z = 2.4952\ \text{GeV} \\
M_W = 80.385\ \text{GeV}, &\quad \Gamma_W = 2.085\ \text{GeV} \\
M_H = 125\ \text{GeV}, &\quad \\
m_e = 0.510998928\ \text{MeV}, &\quad m_\mu = 0.1056583715\ \text{GeV}, \quad m_\tau = 1.77682\ \text{GeV} \\
m_u = 0.06983\ \text{GeV}, &\quad m_c = 1.2\ \text{GeV}, \quad m_t = 173.5\ \text{GeV} \\
m_d = 0.06984\ \text{GeV}, &\quad m_s = 0.15\ \text{GeV}, \quad m_b = 4.6\ \text{GeV} \\
|V_{ud}| = 0.975, &\quad |V_{us}| = 0.222 \\
|V_{cd}| = 0.222, &\quad |V_{cs}| = 0.975 \\
|V_{cb}| = |V_{ts}| = |V_{ub}| &= |V_{td}| = |V_{tb}| = 0\ .
\end{aligned} \tag{1}$$

We work in the constant width scheme and fix the weak mixing angle by $c_w = M_W/M_Z$, $s_w^2 = 1 - c_w^2$. The Z and W boson decay widths given above are used in the LO, NLO and NNLO evaluations of the cross sections. To account for the fact that we are using the constant width approach, we have to adjust the W, Z mass and width input parameters that have been measured in the s-dependent width approach accordingly, as follows [2, 3] ($\gamma_V = \Gamma_V/M_V$):

$$M_V \to \frac{M_V}{\sqrt{1+\gamma_V^2}} \ ; \ \Gamma_V \to \frac{\Gamma_V}{\sqrt{1+\gamma_V^2}}\ .$$

Consequently, the input values for the W, Z masses and widths change to

$$\begin{aligned}
M_Z = 91.1535\ \text{GeV}, &\quad \Gamma_Z = 2.4943\ \text{GeV} \\
M_W = 80.358\ \text{GeV}, &\quad \Gamma_W = 2.084\ \text{GeV}\ .
\end{aligned} \tag{2}$$

The fermion masses only enter through loop contributions to the vector-boson self energies and as regulators of the collinear singularities which arise in the calculation of the QED contribution. The light quark masses are chosen in such a way, that the value for the hadronic five-flavour contribution to the photon vacuum polarization, $\Delta\alpha_{had}^{(5)}(M_Z^2) = 0.027572$ [4], is recovered, which is derived from low-energy e^+e^- data with the help of dispersion relations.

- **Proton PDFs and pQCD scale choices**
To compute the hadronic cross section we use the MSTW2008 [5] set of parton distribution functions. (shall we consider NNPDF3.1 instead?) We take the renormalization scale, μ_r, and the QCD factorization scale, μ_{QCD}, to be the invariant mass of the final-state lepton

pair, i.e. $\mu_r = \mu_{\rm QCD} = M_{l\nu}$ in the W boson case and $\mu_r = \mu_{\rm QCD} = M_{l^+l^-}$ in the Z boson case.

All numerical evaluations of EW corrections require the subtraction of QED initial-state collinear divergences, which is performed using the QED DIS scheme. It is defined analogously to the usual DIS [6] scheme used in QCD calculations, i.e. by requiring the same expression for the leading and next-to-leading order structure function F_2 in deep inelastic scattering, which is given by the sum of the quark distributions. Since F_2 data are an important ingredient in extracting PDFs, the effect of the $\mathcal{O}(\alpha)$ QED corrections on the PDFs should be reduced in the QED DIS scheme. The QED factorization scale is chosen to be equal to the QCD factorization scale, $\mu_{QED} = \mu_{QCD}$. The QCD factorization is performed in the $\overline{\rm MS}$ scheme. The subtraction of the QED initial state collinear divergences is a necessary step to obtain a finite partonic cross section.

- **Renormalization**
  We use the following EW input scheme:
  In the calculation of the tree-level couplings we replace $\alpha(0)$ by the effective coupling $\alpha_{G_\mu} = \sqrt{2} G_\mu M_W^2 (1 - M_W^2/M_Z^2)/\pi$. The relative $\mathcal{O}(\alpha)$ corrections are calculated with the fine structure constant $\alpha(0)$. At NLO EW this replacement implies an additional contribution of Δr to the relative $\mathcal{O}(\alpha)$ corrections. The one-loop result for Δr has been calculated in Refs. [7, 8] and can be decomposed as follows:

$$\Delta r(1-\text{loop}) = \Delta\alpha - \frac{c_w^2}{s_w^2}\Delta\rho + \Delta r_{rem}(M_H) \ .$$

When using the input values of Eq. 1 and the values for M_W and M_Z given in item ??? $\Delta r(1-\text{loop}) = 0.0295633444$ ($\Delta r = 0.0296123554$ for the unshifted W/Z masses of Eq. 1).

For NLO EW predictions, we work in the on-shell renormalization scheme and use the following Z and W mass renormalization constants:

$$\delta M_Z^2 = \mathcal{R}e\Sigma^Z(M_Z^2), \quad \delta M_W^2 = \mathcal{R}e\Sigma^W(M_W^2) \ , \tag{3}$$

where Σ^V denotes the transverse part of the unrenormalized vector-boson self energy.

In the course of the calculation of radiative corrections to W boson observables the Kobayashi-Maskawa mixing has been neglected, but the final result for each parton level process has been multiplied with the square of the corresponding physical matrix element V_{ij}. From a numerical point of view, this procedure does not significantly differ from a consideration of the Kobayashi-Maskawa matrix in the renormalisation procedure as it has been pointed out in [9].

We choose to evaluate the running of the strong coupling constant at the two-loop level, with five flavours, for LO, NLO and NLO+PS predictions using as reference value $\alpha_s^{NLO}(M_Z) = 0.12018$, which is consistent with the choice made in the NLO PDF set of MSTW2008. For the NNLO QCD predictions we use the NNLO PDF set and correspondingly the three-loop running of $\alpha_s(\mu_r)$, with reference value $\alpha_s^{NNLO}(M_Z) = 0.117$. In Table 1 we provide $\alpha_s(\mu_r^2)$ for several choices of the QCD renormalization scale μ_r, which are consistent with the results provided by the LHAPDF function `alphasPDF(`μ_r`)` when called in conjunction with MSTW2008. (shall we update consistently with the NNPDF3.1 set?)

Table 1: Two-loop and three-loop running of $\alpha_s(\mu_r^2)$.

μ_r [GeV]	α_s(NLO)	α_s(NNLO)
91.1876	0.1201789	0.1170699
50	0.1324396	0.1286845
100	0.1184991	0.1154741
200	0.1072627	0.1047716
500	0.0953625	0.0933828

- **Acceptance cuts**
  The detector acceptance is simulated by imposing the following transverse momentum ($p_\perp$) and pseudo-rapidity (η) cuts:

$$\begin{aligned} \text{LHC}: &\quad p_\perp^\ell > 25 \text{ GeV}, \quad |\eta(\ell)| < 2.5,\ p_\perp^\nu > 25 \text{ GeV}, \quad \ell = e,\, \mu, \\ \text{LHCb}: &\quad p_\perp^\ell > 20 \text{ GeV}, 2 < \eta(\ell) < 4.5,\ p_\perp^\nu > 20 \text{ GeV}, \quad \ell = e,\, \mu\ , \end{aligned} \tag{4}$$

  where $p_\perp^\nu$ is the missing transverse momentum originating from the neutrino. These cuts approximately model the acceptance of the ATLAS, CMS, and LHCb detectors at the LHC. In addition to the separation cuts of Eq. 4 we apply a cut on the invariant mass of the final-state lepton pair of $M_{l^+l^-} > 50$ GeV and $M(l\nu) > 1$ GeV in the case of γ/Z production and W production respectively. In the case of W boson production, in addition to the acceptance cuts we apply $M_\perp(l\nu) > 40$ GeV.

  Results are provided for the *bare* setup, i.e. when only applying the acceptance cuts of Eq. 4. Since we consider predictions inclusive with respect to QCD radiation, we do not impose any jet definition.

  We use the Pythia version 6.4.26, Perugia tune (PYTUNE(320)). When producing NLO QCD+EW results with Pythia, the QED showering effects are switched off by setting `MSTJ(41)=MSTP(61)=MSTP(71)=1`. (shall we consider a Pythia8 version and tune?)

- W boson observables
  - σ_W: total inclusive cross section of W boson production.
  - $\frac{d\sigma}{dM_\perp(l\nu)}$: transverse mass distribution of the lepton lepton-neutrino pair. The transverse mass is defined as
$$M_\perp = \sqrt{2p_\perp^\ell p_\perp^\nu(1-\cos\phi^{\ell\nu})}\ , \tag{5}$$
where $p_\perp^\nu$ is the transverse momentum of the neutrino, and $\phi^{\ell\nu}$ is the angle between the charged lepton and the neutrino in the transverse plane.
  - $\frac{d\sigma}{dp_\perp^l}$: charged lepton transverse momentum distribution.
  - $\frac{d\sigma}{dp_\perp^\nu}$: missing transverse momentum distribution.
  - $d\sigma_W/dp_\perp^W$: lepton-pair (W) transverse momentum distribution.
- Z boson observables
  - σ_Z: total inclusive cross section of Z boson production.
  - $\frac{d\sigma}{dM_{l^+l^-}}$: invariant mass distribution of the lepton pair.
  - $\frac{d\sigma}{dp_\perp^l}$: transverse lepton momentum distribution (l is the positively charged lepton).
  - $d\sigma_Z/dp_\perp^Z$: lepton-pair (Z) transverse momentum distribution.
  - Finally, for the case of Z boson production we add the distribution in ϕ^* to our list of observables. This observable is defined, e.g., in Ref. [10] as follows:
$$\phi^* = \tan\left(\frac{\pi-\Delta\Phi}{2}\right)\sin(\theta_\eta^*)\ ,$$
with $\Delta\Phi = \Phi^- - \Phi^+$ denoting the difference in the azimuthal angle of the two negatively/positively charged leptons in the laboratory frame, and
$$\cos(\theta_\eta^*) = \tanh\left(\frac{\eta^- - \eta^+}{2}\right)\ .$$
$\eta^\pm$ denote the pseudo rapidity of the negatively/positively charged lepton.
  - Angular coefficients.

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