Proposal for a benchmark for $H_5^{0,\pm,\pm\pm}$ searches in the Georgi-Machacek model

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Abstract: In this note I propose a benchmark plane for LHC searches for the custodial fiveplet states in the Georgi-Machacek model, following the call by the LHC Higgs Cross Section Working Group.

1 Introduction

One of the most interesting features of the Georgi-Machacek (GM) model [1, 2] is the presence of a custodial fiveplet of Higgs bosons,

$$H_5^{++}, H_5^{+}, H_5^{0}, H_5^{-}, H_5^{--}, \tag{1}$$

that couple to W and Z boson pairs at tree level. These states offer interesting signatures at the LHC, such as vector boson fusion (VBF) production of $H_5^{\pm}$ followed by decays to $W^{\pm}Z$, which has already been studied by ATLAS in Run 1 [3].

In this note I propose a benchmark plane in which the free parameters most relevant for the H_5 searches are varied. The plane is designed to be compatible with the spectrum calculator `GMCALC` [5]. This benchmark provides an explicit realization of the recommendations for H_5 searches in VBF given in Ref. [4].

2 Model parameterization

The scalar sector of the GM model [1, 2] consists of the usual complex isospin doublet (ϕ^+, ϕ^0) with hypercharge[1] $Y = 1$, a real triplet (ξ^+, ξ^0, ξ^-) with $Y = 0$, and a complex triplet $(\chi^{++}, \chi^+, \chi^0)$ with $Y = 2$. The doublet is responsible for the fermion masses as in the SM.

The scalar potential is chosen by hand to preserve a global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry. This ensures $\rho = 1$ at tree level. In order to make the global $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ symmetry explicit, we

[1] We normalize the hypercharge operator such that $Q = T^3 + Y/2$.

write the doublet in the form of a bidoublet Φ and combine the triplets to form a bitriplet X:

$$\Phi = \left(\begin{array}{cc} \phi^{0*} & \phi^+ \\ -\phi^{+*} & \phi^0 \end{array} \right), \qquad X = \left(\begin{array}{ccc} \chi^{0*} & \xi^+ & \chi^{++} \\ -\chi^{+*} & \xi^0 & \chi^+ \\ \chi^{++*} & -\xi^{+*} & \chi^0 \end{array} \right). \tag{2}$$

The vacuum expectation values (vevs) are defined by $\langle \Phi \rangle = \frac{v_\phi}{\sqrt{2}} \mathbf{I}_{2\times 2}$ and $\langle X \rangle = v_\chi \mathbf{I}_{3\times 3}$, where $\mathbf{I}$ is the unit matrix. The Fermi constant G_F fixes the combination of vevs,

$$v_\phi^2 + 8 v_\chi^2 \equiv v^2 = \frac{1}{\sqrt{2} G_F} \approx (246 \text{ GeV})^2. \tag{3}$$

The most general gauge-invariant scalar potential involving these fields that conserves custodial SU(2) is given, in the conventions of Ref. [6], by

$$\begin{aligned} V(\Phi, X) &= \frac{\mu_2^2}{2} \mathrm{Tr}(\Phi^\dagger \Phi) + \frac{\mu_3^2}{2} \mathrm{Tr}(X^\dagger X) + \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 + \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi) \mathrm{Tr}(X^\dagger X) \\ &\quad + \lambda_3 \mathrm{Tr}(X^\dagger X X^\dagger X) + \lambda_4 [\mathrm{Tr}(X^\dagger X)]^2 - \lambda_5 \mathrm{Tr}(\Phi^\dagger \tau^a \Phi \tau^b) \mathrm{Tr}(X^\dagger t^a X t^b) \\ &\quad - M_1 \mathrm{Tr}(\Phi^\dagger \tau^a \Phi \tau^b)(U X U^\dagger)_{ab} - M_2 \mathrm{Tr}(X^\dagger t^a X t^b)(U X U^\dagger)_{ab}. \end{aligned} \tag{4}$$

Here the SU(2) generators for the doublet representation are $\tau^a = \sigma^a/2$ with σ^a being the Pauli matrices, the generators for the triplet representation are

$$t^1 = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{array} \right), \qquad t^2 = \frac{1}{\sqrt{2}} \left(\begin{array}{ccc} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{array} \right), \qquad t^3 = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{array} \right), \tag{5}$$

and the matrix U, which rotates X into the Cartesian basis, is given by [7]

$$U = \left(\begin{array}{ccc} -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ -\frac{i}{\sqrt{2}} & 0 & -\frac{i}{\sqrt{2}} \\ 0 & 1 & 0 \end{array} \right). \tag{6}$$

We decompose the neutral fields into real and imaginary parts according to

$$\phi^0 \to \frac{v_\phi}{\sqrt{2}} + \frac{\phi^{0,r} + i \phi^{0,i}}{\sqrt{2}}, \qquad \chi^0 \to v_\chi + \frac{\chi^{0,r} + i \chi^{0,i}}{\sqrt{2}}, \qquad \xi^0 \to v_\chi + \xi^0. \tag{7}$$

The physical fields can then be organized by their transformation properties under the custodial SU(2) symmetry into a fiveplet, a triplet, and two singlets. The custodial-fiveplet states are given by

$$H_5^{++} = \chi^{++}, \qquad H_5^+ = \frac{(\chi^+ - \xi^+)}{\sqrt{2}}, \qquad H_5^0 = \sqrt{\frac{2}{3}} \xi^0 - \sqrt{\frac{1}{3}} \chi^{0,r}. \tag{8}$$

Because the states in the custodial fiveplet contain no doublet field content, they do not couple to fermions.

The custodial-triplet states are given by

$$H_3^+ = -s_H \phi^+ + c_H \frac{(\chi^+ + \xi^+)}{\sqrt{2}}, \qquad H_3^0 = -s_H \phi^{0,i} + c_H \chi^{0,i}, \tag{9}$$

where the vevs are parameterized by

$$c_H \equiv \cos\theta_H = \frac{v_\phi}{v}, \qquad s_H \equiv \sin\theta_H = \frac{2\sqrt{2}\, v_\chi}{v}. \tag{10}$$

The quantity s_H^2 represents the fraction of the gauge boson masses-squared M_W^2 and M_Z^2 that is generated by the vev of the triplets, while c_H^2 represents the fraction generated by the usual Higgs doublet.

The states of the custodial fiveplet $(H_5^{\pm\pm}, H_5^\pm, H_5^0)$ have a common mass m_5 and the states of the custodial triplet $(H_3^\pm, H_3^0)$ have a common mass m_3. These masses can be written (after eliminating μ_2^2 and μ_3^2 in favor of the vevs) as[2]

$$\begin{aligned} m_5^2 &= \frac{M_1}{4v_\chi} v_\phi^2 + 12 M_2 v_\chi + \frac{3}{2}\lambda_5 v_\phi^2 + 8\lambda_3 v_\chi^2, && (12) \\ m_3^2 &= \frac{M_1}{4v_\chi}(v_\phi^2 + 8v_\chi^2) + \frac{\lambda_5}{2}(v_\phi^2 + 8v_\chi^2) = \left(\frac{M_1}{4v_\chi} + \frac{\lambda_5}{2}\right) v^2. && (13) \end{aligned}$$

The two custodial-singlet mass eigenstates are given by

$$h = \cos\alpha\, \phi^{0,r} - \sin\alpha\, H_1^{0\prime}, \qquad H = \sin\alpha\, \phi^{0,r} + \cos\alpha\, H_1^{0\prime}, \tag{14}$$

where

$$H_1^{0\prime} = \sqrt{\frac{1}{3}}\xi^0 + \sqrt{\frac{2}{3}}\chi^{0,r}. \tag{15}$$

The mixing angle and masses are given by

$$\begin{aligned} \sin 2\alpha &= \frac{2\mathcal{M}_{12}^2}{m_H^2 - m_h^2}, \qquad \cos 2\alpha = \frac{\mathcal{M}_{22}^2 - \mathcal{M}_{11}^2}{m_H^2 - m_h^2}, && (16) \\ m_{h,H}^2 &= \frac{1}{2}\left[\mathcal{M}_{11}^2 + \mathcal{M}_{22}^2 \mp \sqrt{\left(\mathcal{M}_{11}^2 - \mathcal{M}_{22}^2\right)^2 + 4\left(\mathcal{M}_{12}^2\right)^2}\right], && (17) \end{aligned}$$

[2]Note that the ratio M_1/v_χ can be written using the minimization condition $\partial V/\partial v_\chi = 0$ as

$$\frac{M_1}{v_\chi} = \frac{4}{v_\phi^2}\left[\mu_3^2 + (2\lambda_2 - \lambda_5)v_\phi^2 + 4(\lambda_3 + 3\lambda_4)v_\chi^2 - 6M_2 v_\chi\right], \tag{11}$$

which is finite in the limit $v_\chi \to 0$.

where we choose $m_h < m_H$, and

$$\begin{aligned}
\mathcal{M}_{11}^2 &= 8\lambda_1 v_\phi^2, \\
\mathcal{M}_{12}^2 &= \frac{\sqrt{3}}{2} v_\phi \left[-M_1 + 4\left(2\lambda_2 - \lambda_5\right) v_\chi\right], \\
\mathcal{M}_{22}^2 &= \frac{M_1 v_\phi^2}{4 v_\chi} - 6 M_2 v_\chi + 8\left(\lambda_3 + 3\lambda_4\right) v_\chi^2.
\end{aligned} \tag{18}$$

The fiveplet states couple to vector bosons according to the following Feynman rules [8, 9, 6]:

$$\begin{aligned}
H_5^0 W_\mu^+ W_\nu^- :&\quad \sqrt{\frac{2}{3}} i g^2 v_\chi g_{\mu\nu} = 2i \frac{M_W^2}{v}\left(\frac{s_H}{\sqrt{3}}\right) g_{\mu\nu} = 2(\sqrt{2}G_F)^{1/2} M_W^2 \left(-\frac{s_H}{\sqrt{3}}\right)(-i g_{\mu\nu}), \\
H_5^0 Z_\mu Z_\nu :&\quad -\sqrt{\frac{8}{3}} i \frac{g^2 v_\chi}{c_W^2} g_{\mu\nu} = 2i \frac{M_Z^2}{v}\left(-\frac{2 s_H}{\sqrt{3}}\right) g_{\mu\nu} = 2(\sqrt{2}G_F)^{1/2} M_Z^2 \left(\frac{2 s_H}{\sqrt{3}}\right)(-i g_{\mu\nu}), \\
H_5^+ W_\mu^- Z_\nu :&\quad -\sqrt{2} i \frac{g^2 v_\chi}{c_W} g_{\mu\nu} = 2i \frac{M_W M_Z}{v}\left(-s_H\right) g_{\mu\nu} = 2(\sqrt{2}G_F)^{1/2} M_W M_Z \left(s_H\right)(-i g_{\mu\nu}), \\
H_5^{++} W_\mu^- W_\nu^- :&\quad 2 i g^2 v_\chi g_{\mu\nu} = 2i \frac{M_W^2}{v}\left(\sqrt{2} s_H\right) g_{\mu\nu} = 2(\sqrt{2}G_F)^{1/2} M_W^2 \left(-\sqrt{2} s_H\right)(-i g_{\mu\nu}),
\end{aligned} \tag{19}$$

where we write the coupling in multiple forms to make contact with the notation of Refs. [9, 10]. The triplet vev v_χ is called v' in Ref. [9], and the factors F_{VV} in Eq. (5.2) of Ref. [10] correspond in this model to

$$\begin{aligned}
F_{W^+W^-} &= -\frac{s_H}{\sqrt{3}} && (H_5^0 \text{ production}), \\
F_{ZZ} &= \frac{2 s_H}{\sqrt{3}} && (H_5^0 \text{ production}), \\
F_{W^\pm Z} &= s_H && (H_5^\pm \text{ production}), \\
F_{W^\pm W^\pm} &= -\sqrt{2} s_H && (H_5^{\pm\pm} \text{ production}).
\end{aligned} \tag{20}$$

Note in particular that, for H_5^0, one cannot simply rescale the vector boson fusion cross section of the SM Higgs boson because the ratio of WW to ZZ couplings is different than in the SM (for SM Higgs production, $F_{W^+W^-} = F_{ZZ} = -1$).

All the $H_5 VV$ couplings are proportional to s_H, so that the VBF and $q\bar{q}^{(\prime)} \to V^* \to V^{(\prime)} H_5$ production cross sections of the H_5 states, as well as their decay widths into VV, are proportional to s_H^2. In contrast, cross sections for associated production of two H_5 states, $q\bar{q}^{(\prime)} \to V^* \to H_5 H_5^\prime$, are controlled by gauge couplings and thus depend only on m_5, not on s_H.

3 Benchmark proposal

We propose the "H5plane" benchmark as follows. We choose as free parameters m_h, m_5, s_H, λ_2, λ_3, λ_4, M_1, and M_2, i.e., the input parameters of `INPUTSET = 4` in `GMCALC`:

$$\begin{aligned} m_h &= 125 \text{ GeV} \\ m_5 &\in [200, 2000] \text{ GeV} \\ s_H &\in (0, 1) \\ \lambda_2 &= 0.4(m_5/1000 \text{ GeV}) \\ \lambda_3 &= -0.1 \\ \lambda_4 &= 0.2 \\ M_1 &= \frac{\sqrt{2} s_H}{v}(m_5^2 + v^2) \\ M_2 &= M_1/6. \end{aligned} \tag{21}$$

Figure 1 shows the theoretically-accessible region in the m_5–s_H plane in the full GM model (red) and in the H5plane benchmark (black). This plot was generated using `GMCALC 1.1.1` with $m_h = 125$ GeV and using `INPUTSET = 4`. All theoretical constraints were imposed, i.e., perturbative unitarity of the scalar quartic couplings, bounded-from-belowness of the scalar potential, and the absence of deeper alternative minima. No experimental constraints were imposed in the parameter scans. We also show the region excluded by the cross section for like-sign W boson pair production in VBF from Ref. [11] (above and to the left of the blue line).

This benchmark has the following features:

- It fully populates the theoretically-allowed region of the m_5–s_H plane for $m_5 \in [200, 2000]$ GeV, except for a small corner at low m_5 and high s_H which is already excluded by the cross section for like-sign W boson pair production in VBF [11] (this limits the maximum allowed production cross section for VBF$\to H_5^{\pm\pm} \to W^\pm W^\pm$, and hence sets an upper bound on s_H as a function of m_5).

- The "loose" constraint from $b \to s\gamma$ as defined in Ref. [12] eliminates only points that are already excluded by the cross section for like-sign W boson pair production in VBF [11].

- The benchmark is not unreasonably constrained by coupling measurements of the 125 GeV Higgs: the region of the m_5–s_H plane in which $|\kappa_i^h - 1| < 0.1$, with $i = f, V, \gamma$, is essentially the same in the H5plane benchmark as in the full parameter scan.

- It has $m_3 \gtrsim m_5 + 10$ GeV over the whole benchmark plane, so that the Higgs-to-Higgs decays $H_5 \to H_3 H_3$ and $H_5 \to H_3 V$ are kinematically forbidden, leaving only the decays $H_5 \to VV$ at tree level; i.e., $\text{BR}(H_5 \to VV) = 1$ to a very good approximation.

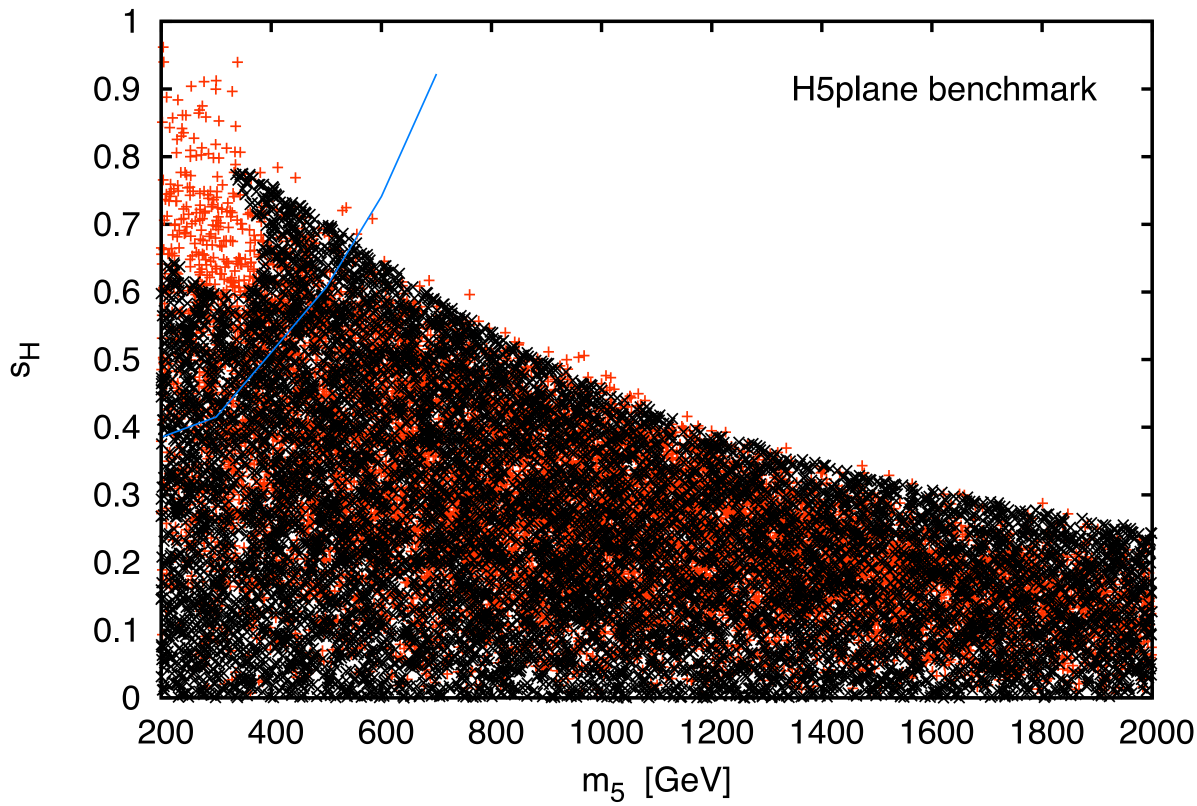

Figure 1: Theoretically-accessible region in the m_5–s_H plane from a full parameter scan (red) and in the H5plane benchmark (black). The region above and to the left of the solid blue curve is excluded by the cross section for like-sign W boson pair production in VBF as determined in Ref. [11].

- It has $m_H \gtrsim m_5 + 12$ GeV over the whole benchmark plane, except for a few points at $s_H > 0.7$ which are already excluded by the cross section for like-sign W boson pair production in VBF [11]. However, there is a large region of parameter space covering $m_5 \gtrsim 600$ GeV and $0.07 \lesssim s_H \lesssim 0.6$ in which the total decay widths of H_5^0 and H are larger than the mass splitting between these two states. In this region, a dedicated study of the lineshape and interference effects of the two resonances in VBF$\to (H_5^0, H) \to WW, ZZ$ will be required.

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