June 2011 — CERN-PH-TH/2011-134 – FR-PHENO-2011-009 – TTK-11-17 – WUB/11-04

Bottom-quark associated Higgs-boson production: reconciling the four- and five-flavour scheme approach

R. Harlander[1,2], M. Krämer[3], and M. Schumacher[4]

[1]*Fachbereich C, Bergische Universität Wuppertal, 42097 Wuppertal, Germany*

[2]*PH-TH, CERN, CH-1211, Geneva 23, Switzerland*

[3]*Institute for Theoretical Particle Physics and Cosmology RWTH Aachen University, D-52056 Aachen, Germany*

[4]*Fakultät für Mathematik und Physik, Albert-Ludwigs Universität Freiburg, D-79104 Freiburg, Germany*

Abstract

The main arguments in the discussion of the proper treatment of the total inclusive cross section for bottom-quark associated Higgs-boson production are briefly reviewed. A simple and pragmatic formula for the combination of the so-called four- and five-flavour schemes is suggested, including the treatment of the respective theory error estimates. The numerical effects of this matching formula are discussed.

1 Two approaches

The inclusive total cross section for bottom-quark associated Higgs-boson production, denoted[1] $pp/p\bar{p} \to (b\bar{b})H + X$, can be calculated in two different schemes. As the mass of the bottom-quark is large compared to the QCD scale, $m_\mathrm{b} \gg \Lambda_\mathrm{QCD}$, bottom-quark production is a perturbative process and can be calculated order by order. Thus, in a four-flavour scheme (4FS), where one does not consider b-quarks as partons in the proton, the lowest-order QCD production processes are gluon-gluon fusion and quark-antiquark annihilation, $gg \to b\bar{b}H$ and $q\bar{q} \to b\bar{b}H$, respectively. However, the inclusive cross section for $gg \to (b\bar{b})H$ develops logarithms of the form $\ln(\mu_\mathrm{F}/m_\mathrm{b})$, which arise from the splitting of gluons into nearly collinear $b\bar{b}$ pairs. The large factorization scale $\mu_\mathrm{F} \approx m_\mathrm{H}/4$ corresponds to the upper limit of the collinear region up to which factorization is valid [1, 2, 3]. For Higgs-boson masses $m_\mathrm{H} \gg 4m_\mathrm{b}$, the logarithms become large and spoil the convergence of the perturbative series. The $\ln(\mu_\mathrm{F}/m_\mathrm{b})$ terms can be summed to all orders in perturbation theory by introducing bottom parton densities. This defines the so-called five-flavour scheme (5FS). The use of bottom distribution functions is based on the approximation that the outgoing b-quarks are at small transverse momentum. In this scheme, the LO process for the inclusive $(b\bar{b})H$ cross section is bottom fusion, $b\bar{b} \to H$.

If all orders in perturbation theory were taken into account, the four- and five-flavour schemes would be identical, but the way of ordering the perturbative expansion is different.

[1]The notation $(b\bar{b})H$ is meant to indicate that the $b\bar{b}$ pair is not required as part of the signature in this process, so that its final state momenta must be integrated over the full phase space.

At any finite order, the two schemes include different parts of the all-order result, and the cross section predictions do thus not match exactly. While this leads to an ambiguity in the way the cross section is calculated, it also offers an opportunity to test the importance of various higher-order terms and the reliability of the theoretical prediction. The 4FS calculation is available at NLO [4, 5], while the 5FS cross section has been calculated at NNLO accuracy [6]. Electroweak corrections to the 5FS process have been found to be small [7] and will not be considered in the numerical results presented in this note.

In the next section we briefly summarize some of the features of the two schemes.

2 Discussion

In this section, we collect the main arguments and counter-arguments that have been discussed in the physics community for and against either the 4FS or the 5FS approach. Let us stress that at this point, we consider only the total inclusive cross section $pp \to (b\bar{b})H + X$. Once distributions or even tagged b-quarks are taken into account, the discussion below has to be re-evaluated.

Gluon to b-quark splitting — kinematical approximations and collinear logarithms: In pp or $p\bar{p}$ collisions, bottom-quarks arise at LO in α_s through the splitting of a gluon. In the 4FS, this splitting is described by fixed-order perturbation theory, and includes the full dependence on the transverse momentum p_T of the b-quark and its mass. In the 5FS scheme, the splitting arises by solving the DGLAP evolution equations with five massless quark flavours [8].

Integrating over phase space, the 5FS at LO neglects all contributions beyond $1/p_T^2$ as $p_T \to 0$, while they are consistently taken into account in the 4FS. However, at higher orders, the 5FS does include sub-leading finite p_T effects; at NNLO, all the information of the LO 4FS calculation is incorporated in the 5FS as well. Note, however, that the 4FS result is available through NLO, so that the finite p_T effects are available one order higher than in the 5FS calculation.

Since the factorization theorem holds only for massless quarks, in the 5FS scheme one has to assume $m_{\mathrm{b}} = 0$ (the bottom Yukawa coupling can – and must – be kept finite though). As the only other scales of the inclusive cross section are m_{H} and the hadronic center-of-mass energy $s \gg m_{\mathrm{H}}^2$, the 4FS and the 5FS formally differ at order $m_{\mathrm{b}}^2/m_{\mathrm{H}}^2$, even if all orders of the perturbation theory could be taken into account. The numerical size of these effects has been investigated in Ref. [9] and was found to be negligible.

In the opposite phase space region of very small transverse momenta of the b-quarks, the 5FS implicitly re-sums logarithmic terms of order $\alpha_s^{n+2}/p_T^2 \cdot \ln^{2n-k} p_T^2$ $(n \geq 0)$, where $k = 0$ in the LO, $k \in \{0,1\}$ in the NLO, and $k \in \{0,1,2\}$ in the NNLO calculation. In the 4FS, these effects are not re-summed, but taken into account only up to finite n.

Renormalization/factorization scale dependence: Due to the different complexity of the two LO processes ($2 \to 3$ for the 4FS and $2 \to 1$ for the 5FS), radiative corrections are known through NNLO in the 5FS [6], while they are available only through NLO in the 4FS [4, 5]. This results in a stronger dependence in the 4FS on the unphysical renormalization and factorization scales, μ_R and μ_F, than in the 5FS.

Top-quark induced effects: Currently, the 5FS calculation neglects effects proportional to the top Yukawa coupling. Partly, they are taken into account in the gluon fusion induced cross section, $gg \to H + X$. What is missing are interference effects proportional to both the top and the bottom Yukawa coupling. In most models with an enhanced bottom Yukawa coupling (like SUSY at large $\tan\beta$), the top Yukawa coupling is simultaneously suppressed though, and therefore these interference terms can be neglected. For consistency, these terms have been removed also from the 4FS calculations in the numerical results below.

Parton distribution functions: Technically, the transition from n_f to $n_f + 1$ flavours is well understood through NNLO [10]. The exact value $\mu_{\rm thr}$ where this transition is imposed is arbitrary (but fixed in all available PDF sets). At fixed order in perturbation theory, this introduces an uncertainty in the bottom-quark PDFs which should, however, be small for $m_H \sim \mu_F \gg \mu_{\rm thr} \sim m_b$.

In fact, until recently, modern PDF sets always assumed a five-flavour content of the proton. This led to an inconsistency in the 4FS calculation. With the return of four-flavour PDF sets [11, 12, 13], this inconsistency could be removed.

Numerical phase space integration: Phase space integration in the 4FS calculation is done numerically. In the collinear region, this requires high precision and is computationally very demanding. In the 5FS, the analytical result for the phase space integration is published and implemented in a publicly available program `bbh@nnlo` [6] which performs the convolution with the PDFs. The calculation of the cross section for a single set of parameters takes of the order of seconds with `bbh@nnlo`.

Disagreement between the 4FS and 5FS: If the total inclusive cross section is calculated at LO in both the 4FS and 5FS, and the factorization scale is set to $\mu_F = m_H$, one finds that the numerical results disagree by a factor of the order of five (see, e.g., Ref. [14]). As already mentioned in Section 1, it was argued that in the 5FS, the "proper" choice of the factorization scale is $\mu_F \sim m_H/4$. In fact, the 4FS and 5FS results agree very well in this case. The NNLO calculation in the 5FS shows a rather mild dependence on μ_F, and indeed confirms that $\mu_F = m_H/4$ is a reasonable choice, since the radiative corrections exhibit a good convergence behaviour for that value.

3 Santander matching

Theoretically, the 4FS and the 5FS are equivalent descriptions of the total inclusive cross section for $(b\bar{b})H$ production. However, due to the arguments listed above, differences in the numerical predictions are observed at finite order of perturbation theory. By making what we believe are reasonable assumptions, in this section we suggest a way to combine both approaches.

The 4FS and 5FS calculations provide the unique description of the cross section in the asymptotic limits $m_H/m_b \to 1$ and $m_H/m_b \to \infty$, respectively. For phenomenologically relevant Higgs-boson masses away from these asymptotic regions both schemes are applicable and include different types of higher-order contributions. The matching we suggest below interpolates between the asymptotic limits of very light and very heavy Higgs-bosons. It is pragmatic and far from being theoretically rigorous. However, we believe that the matching catches the essence of the arguments given in Section 2 and provides a prediction with an uncertainty band which covers the physical result.

A comparison of the 4FS and 5FS calculations reveals that both are in numerical agreement for moderate Higgs-boson masses (see Fig. 23 of Ref. [15]). Once larger Higgs-boson masses are considered, the effect of the collinear logarithms $\ln(m_{\rm H}/m_{\rm b})$ becomes more and more important and the two approaches begin to differ. We suggest to combine the two approaches in such a way that they are given variable weight, depending on the value of the Higgs-boson mass.

The difference between the two approaches is formally logarithmic. Therefore, the dependence of their relative importance on the Higgs-boson mass should be controlled by a logarithmic term. We determine the coefficients such that

(a) the 5FS gets 100% weight in the limit $m_{\rm H}/m_{\rm b} \to \infty$;

(b) the 4FS gets 100% weight in the limit where the logarithms are "small". There is obviously quite some arbitrariness in this statement. We assume here that "small" means $\ln(m_{\rm H}/m_{\rm b}) = 2$. The consequence of this particular choice is that the 4FS and the 5FS both get the same weight for Higgs-boson masses around 100 GeV, consistent with the observed agreement between the 4FS and the 5FS in this region.[2]

This leads to the following formula[3]

$$\sigma^{\rm matched} = \frac{\sigma^{\rm 4FS} + w\,\sigma^{\rm 5FS}}{1+w}\,, \tag{1}$$

with the weight w defined as

$$w = \ln\frac{m_{\rm H}}{m_{\rm b}} - 2\,, \tag{2}$$

and $\sigma^{\rm 4FS}$ and $\sigma^{\rm 5FS}$ denote the total inclusive cross section in the 4FS and the 5FS, respectively. For $m_{\rm b} = 4.75\,{\rm GeV}$ and specific values of $m_{\rm H}$, this leads to

$$\begin{aligned}
\sigma^{\rm matched}\big|_{m_{\rm H}=100\,{\rm GeV}} &= 0.49\,\sigma^{\rm 4FS} + 0.51\,\sigma^{\rm 5FS}\,,\\
\sigma^{\rm matched}\big|_{m_{\rm H}=200\,{\rm GeV}} &= 0.36\,\sigma^{\rm 4FS} + 0.64\,\sigma^{\rm 5FS}\,,\\
\sigma^{\rm matched}\big|_{m_{\rm H}=300\,{\rm GeV}} &= 0.31\,\sigma^{\rm 4FS} + 0.69\,\sigma^{\rm 5FS}\,,\\
\sigma^{\rm matched}\big|_{m_{\rm H}=400\,{\rm GeV}} &= 0.29\,\sigma^{\rm 4FS} + 0.71\,\sigma^{\rm 5FS}\,,\\
\sigma^{\rm matched}\big|_{m_{\rm H}=500\,{\rm GeV}} &= 0.27\,\sigma^{\rm 4FS} + 0.73\,\sigma^{\rm 5FS}\,.
\end{aligned} \tag{3}$$

A graphical representation of the weight factor w is shown in Fig. 1 (a).

Concerning the uncertainties, we suggest to add them linearly, using the weights w defined in Eq. (2). This ensures that the combined error is always larger than the minimum of the

[2] Note that one should use the *pole mass* for $m_{\rm b}$ here rather than the running mass, since it is really the dynamical mass that rules the re-summed logarithms.

[3] This formula originated from discussions among the authors at the *Higgs Days at Santander 2009* and is therefore dubbed "Santander matching".

two individual errors. Neglecting correlations and assuming equality of the uncertainties in the 4FS and 5FS calculations would imply that the matched uncertainty is reduced by a factor of $w/(1+w)$ with respect to the common individual uncertainties. This seems unreasonable. In our approach the matched uncertainty would be equal to the individual ones in this case. On the other hand, taking the envelope of the 4FS and 5FS error bands seems overly conservative to us.

The theoretical uncertainties in the 4FS and the 5FS calculations are obtained through $\mu_{\rm F}$-, $\mu_{\rm R}$-, PDF-, and α_s variation as described in Ref. [15]. They can be quite asymmetric, which is why the combination should be done separately for the upper and the lower uncertainty limits:

$$\Delta\sigma_\pm = \frac{\Delta\sigma_\pm^{\rm 4FS} + w\,\Delta\sigma_\pm^{\rm 5FS}}{1+w}\,, \tag{4}$$

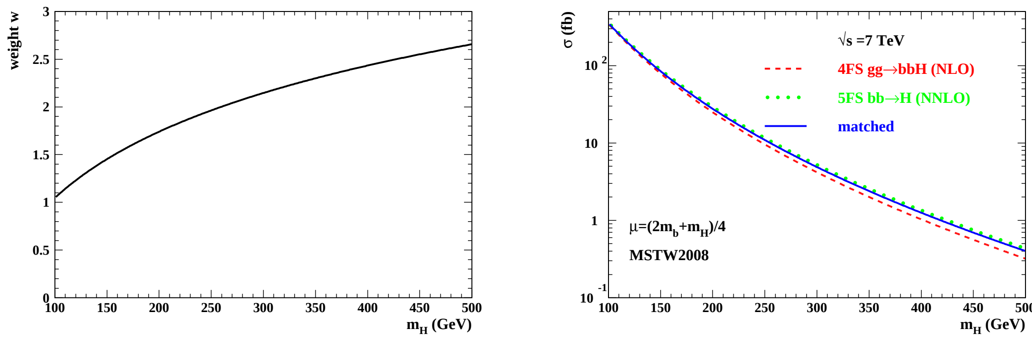
where $\Delta\sigma_\pm^{\rm 4FS}$ and $\Delta\sigma_\pm^{\rm 5FS}$ are the upper/lower uncertainty limits of the 4FS and the 5FS, respectively.

4 Numerical results

We shall discuss the numerical implications of the matching prescriptions defined in the previous section and provide matched predictions for the inclusive cross section $pp \to (b\bar{b})H + X$ at the LHC operating at a center-of-mass energy of 7 TeV. The individual numerical results for the 4FS and 5FS calculations have been obtained in the context of the *LHC Higgs Cross Section Working Group* with input parameters as described in Ref. [15]. Note that the cross section predictions presented below correspond to Standard Model bottom Yukawa couplings and a bottom-quark mass of $m_{\rm b} = 4.75$ GeV. SUSY effects can be taken into account by simply rescaling the bottom Yukawa coupling to the proper value [7, 16].

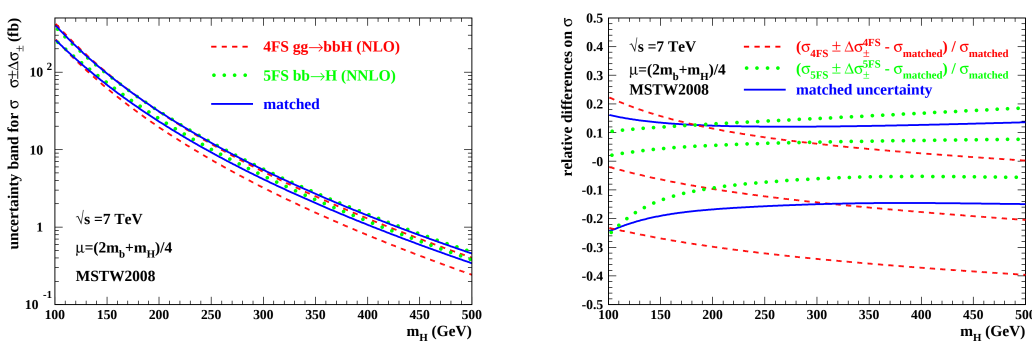
Fig. 1 (b) shows the central values for the 4FS and the 5FS cross section, as well as the matched result, as a function of the Higgs-boson mass. The ratio of the central 4FS and the 5FS predictions to the matched result is displayed in Fig. 2 (b) (central dashed and dotted line): for $m_{\rm H} = 100$ GeV, the 4FS and the 5FS contribute with approximately the same weight to the matched cross section, with deviations between the individual 4FS and the 5FS predictions and the matched one of less than 5%. With increasing Higgs-boson mass, the 4FS result deviates more and more from the matched cross section due to its decreasing weight. At $m_{\rm H} = 500$ GeV, it agrees to less than 20% with the matched result, while the 5FS is still within 8% of the latter.

The corresponding theory error estimates are shown in Fig. 2. The absolute numbers are displayed in panel (a), while in panel (b) they are shown relative to the central value of the matched result. Up to $m_{\rm H} \approx 300$ GeV, the combined uncertainty band covers the central values of both the 4FS and the 5FS. For larger Higgs-boson masses, the 4FS central value is slightly outside this band.



(a) (b)

Figure 1: (a) Weight factor w, Eq. (2), as a function of the Higgs-boson mass m_{H}. The bottom-quark pole mass has been set to $m_{\mathrm{b}} = 4.75$ GeV. (b) Central values for the total inclusive cross section in the 4FS (red, dashed), the 5FS (green, dotted), and for the matched cross section (blue, solid). Here and in the following we use the MSTW2008 PDF set [11] (NLO for 4FS, NNLO for 5FS).



(a) (b)

Figure 2: (a) Theory uncertainty bands for the total inclusive cross section in the 4FS (red, dashed), the 5FS (green, dotted), and for the matched cross section (blue, solid). (b) Uncertainty bands and central values, relative to the central value of the matched result (same line coding as panel (a)).

5 Conclusions

We have reviewed the main arguments for and against following either the 4FS or the 5FS when predicting the value of the total inclusive cross section for bottom-quark associated Higgs-boson production. Based on this discussion, we have suggested a way to combine the two approaches such that their central values and uncertainties enter with appropriate weights.

Acknowledgments. Our thanks go to M. Warsinsky for collaboration, discussion, and comments, and in particular for producing the n-tuples which were used for the numerics in this note; to S. Dittmaier, M. Spira, and W. Kilgore for collaboration on the original 4FS and 5FS calculations on which the results in this note are based; to the LHC Higgs Cross Section Working Group for ideological support; and to S. Heinemeyer for organizing the "Higgs Days at Santander". All of the authors are supported by the Helmholtz Alliance "Physics at the Terascale". In addition, MK is supported by the DFG SFB/TR9 "Computational Particle Physics", RH by BMBF grant 05H09PXE, and MS by BMBF grant 05H09VFC (Forschungsschwerpunkt FSP101 ATLAS-Experiment "Physik auf der TeV-Skala am Large Hadron Collider"). RH would like to thank the CERN PH-TH group for their hospitality.

References

[1] D. Rainwater, M. Spira, D. Zeppenfeld, *Higgs boson production at hadron colliders: Signal and background processes*, [hep-ph/0203187].

[2] T. Plehn, *Charged Higgs boson production in bottom-gluon fusion*, *Phys. Rev.* **D 67** (2003) 014018, [hep-ph/0206121].

[3] F. Maltoni, Z. Sullivan, S. Willenbrock, *Higgs-boson production via bottom-quark fusion*, *Phys. Rev.* **D 67** (2003) 093005, [hep-ph/0301033].

[4] S. Dittmaier, M. Krämer, M. Spira, *Higgs radiation off bottom quarks at the Tevatron and the LHC*, *Phys. Rev.* **D 70** (2004) 074010, [hep-ph/0309204].

[5] S. Dawson, C.B. Jackson, L. Reina, D. Wackeroth, *Exclusive Higgs boson production with bottom quarks at hadron colliders*, *Phys. Rev.* **D 69** (2004) 074027, [hep-ph/0311067].

[6] R.V. Harlander and W.B. Kilgore, *Higgs boson production in bottom quark fusion at next-to-next-to-leading order*, *Phys. Rev.* **D 68** (2003) 013001, [hep-ph/0304035].

[7] S. Dittmaier, M. Krämer, A. Mück, T. Schlüter, *MSSM Higgs-boson production in bottom-quark fusion: Electroweak radiative corrections*, *JHEP* **0703** (2007) 114, [hep-ph/0611353].

[8] R.M. Barnett, H.E. Haber, D.E. Soper, *Ultraheavy particle production from heavy partons at hadron colliders*, *Nucl. Phys.* **B 306** (1988) 697.

[9] C. Buttar *et al.*, *Les Houches physics at TeV colliders 2005, standard model, QCD, EW, and Higgs working group: Summary report*, [hep-ph/0604120].

[10] A. Chuvakin, J. Smith, W.L. van Neerven, *Comparison between variable flavor number schemes for charm quark electroproduction*, *Phys. Rev.* **D 61** (2000) 096004, [hep-ph/9910250].

[11] A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt, *Parton distributions for the LHC*, *Eur. Phys. J.* **C 63** (2009) 189, [arXiv:0901.0002].

[12] H.-L. Lai *et al.*, *New parton distributions for collider physics*, *Phys. Rev.* **D 82** (2010) 074024, [arXiv:1007.2241].

[13] R.D. Ball *et al.*, *Impact of Heavy Quark Masses on Parton Distributions and LHC Phenomenology*, *Nucl. Phys.* **B 849** (2011) 296, [arXiv:1101.1300].

[14] M. Krämer, *Associated Higgs production with bottom quarks at hadron colliders*, [hep-ph/0407080].

[15] S. Dittmaier *et al.* [LHC Higgs Cross Section Working Group Collaboration], *Handbook of LHC Higgs Cross Sections: 1. Inclusive Observables*, [arXiv:1101.0593].

[16] S. Dawson, C.B. Jackson, P. Jaiswal, *SUSY QCD Corrections to Higgs-b Production: Is the Δm_b Approximation Accurate?*, [arXiv:1104.1631].