Memorandum

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Higgs Couplings to Quarks

The relation between the quark pole mass M_Q and the $\overline{\rm MS}$ mass $\overline{m}_Q(M_Q)$ at the scale of the quark pole mass is given by (displayed up to NLO for simplicity) [1]

$$\overline{m}_Q(M_Q) = \frac{M_Q}{1 + \frac{4}{3}\frac{\alpha_s(M_Q)}{\pi}} + \mathcal{O}(\alpha_s^2) \tag{1}$$

while the running of the $\overline{\rm MS}$ mass $\overline{m}_Q(\mu)$ is described by[1] [2]

$$\overline{m}_Q(\mu) = \overline{m}_Q(M_Q)\frac{c[\alpha_s(\mu)/\pi]}{c[\alpha_s(M_Q)/\pi]} \tag{2}$$

with the coefficient function

$$\begin{aligned}
c(x) &= \left(\frac{9}{2}x\right)^{\frac{4}{9}}\left[1 + 0.895\,x + \mathcal{O}(x^2)\right] && \text{for } M_s < \mu < M_c && (3)\\
c(x) &= \left(\frac{25}{6}x\right)^{\frac{12}{25}}\left[1 + 1.014\,x + \mathcal{O}(x^2)\right] && \text{for } M_c < \mu < M_b && (4)\\
c(x) &= \left(\frac{23}{6}x\right)^{\frac{12}{23}}\left[1 + 1.175\,x + \mathcal{O}(x^2)\right] && \text{for } M_b < \mu < M_t && (5)
\end{aligned}$$

Using these mass definitions the corresponding Yukawa couplings can be defined,

$$\begin{aligned}
\bar{g}_Q(M_H) &= \sqrt{2}\,\frac{\overline{m}_Q(M_H)}{v} && (6)\\
\bar{g}_Q(M_Q) &= \sqrt{2}\,\frac{\overline{m}_Q(M_Q)}{v} && (7)\\
g_Q^{pole} &= \sqrt{2}\,\frac{M_Q}{v} && (8)
\end{aligned}$$

with the vacuum expectation value $v = 1/\sqrt{\sqrt{2}G_F}$.

Starting from the (most accurate) RG-improved result for the Higgs decay width into quarks for large Higgs masses (displayed up to NLO for simplicity) [3]

$$\Gamma(H \to Q\bar{Q}) = \bar{g}_Q^2(M_H)\,\frac{3M_H}{16\pi}\left\{1 + \frac{17}{3}\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2)\right\} \tag{9}$$

[1] Note that the $\overline{\rm MS}$ mass $\overline{m}_Q(\overline{m}_Q)$ can be derived from Eq. (2) by evolution from M_Q down to $\overline{m}_Q(\overline{m}_Q)$.

the following (resummed) expressions in terms of the Yukawa couplings $\bar{g}_Q(M_Q)$ and g_Q^{pole} can be derived as

$$\Gamma(H \to Q\bar{Q}) = \bar{g}_Q^2(M_Q) \frac{3M_H}{16\pi} \left\{ \frac{c[\alpha_s(M_H)/\pi]}{c[\alpha_s(M_Q)/\pi]} \right\}^2 \left\{ 1 + \frac{17}{3}\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\} \tag{10}$$

$$= \left(g_Q^{pole}\right)^2 \frac{3M_H}{16\pi} \left\{ \frac{1}{1+\frac{4}{3}\frac{\alpha_s(M_Q)}{\pi}} \frac{c[\alpha_s(M_H)/\pi]}{c[\alpha_s(M_Q)/\pi]} \right\}^2 \left\{ 1 + \frac{17}{3}\frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\} \tag{11}$$

The factors in front of the last bracket account for the non-linearity between the observable $\Gamma(H \to Q\bar{Q})$ and the squared Yukawa coupling, while keeping *strict linearity* between the Yukawa coupling and the corresponding quark mass definition.

Expanding Eqs. (10, 11) up to NLO the well-known results are obtained,

$$\Gamma(H \to Q\bar{Q}) = \frac{3G_F M_H}{4\sqrt{2}\pi} \overline{m}_Q^2(M_Q) \left\{ 1 + \left[\frac{17}{3} - 2\log\frac{M_H^2}{M_Q^2} \right] \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\} \tag{12}$$

$$= \frac{3G_F M_H}{4\sqrt{2}\pi} M_Q^2 \left\{ 1 + \left[3 - 2\log\frac{M_H^2}{M_Q^2} \right] \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\} \tag{13}$$

However, by using Eqs. (9, 10, 11) for the experimental fits the same theoretical uncertainties can be assumed[2] so that the fits for the Yukawa couplings $\bar{g}_Q(M_H)$, $\bar{g}_Q(M_Q)$ and g_Q^{pole} can be performed at the same level of accuracy while keeping the strict linearity between the Yukawa coupling and the quark mass. These theoretical features persist at higher orders, but they become more involved due to the admixture of contributions involving different couplings than the Yukawa coupling of interest. This is valid for the electroweak corrections (including QED corrections as a subclass) as well[3].

The basic drawback of the present method [5] to fit the Higgs couplings is that it is only working approximately. This method uses different mass definitions in different observables and neglects the mixing between different couplings in the electroweak corrections and the QCD corrections beyond NLO, while rescaling the LO-inspired couplings by common factors for all observables involved in the global fit. However, non-linearity between the Higgs couplings and the masses must not be confused with non-linearity caused by an inconsistent/approximate scheme for the fits.

Testing Higgs Couplings

The simplified and approximate interim recommendation to fit the Higgs couplings experimentally is based on rescaling factors κ_i for each Higgs Yukawa coupling and the Higgs

[2]The pole mass M_Q is plagued by additional infrared renormalon ambiguities of about $100 - 200$ MeV [4] which have to be added to the theoretical uncertainties of Eq. (9) for the expression Eq. (11) in terms of the quark pole mass.

[3]The proper treatment of electroweak corrections with running fermion masses in general is an open theoretical issue.

couplings to W and Z bosons. Assuming that these factors κ_i $(i = f, V)$ are independent of the chosen scheme for the masses and neglecting admixtures of other Higgs couplings at higher orders in the perturbative expansion of the physical cross sections and decay widths, they can be determined e.g. from the partial decay widths in the following way,

$$\Gamma_{exp}(H \to f\bar{f}) = \kappa_f^2 \Gamma_{SM}(H \to f\bar{f}) \quad (14)$$

$$\Gamma_{exp}(H \to V^* V^{(*)} \to 4f) = \kappa_V^2 \Gamma_{SM}(H \to V^* V^{(*)} \to 4f) \quad (15)$$

Due to these relations the factors κ_i $(i = f, V)$ can be absorbed into effective Higgs couplings to fermions and gauge bosons. The Yukawa couplings imply a linear relation to the corresponding fermion mass, while the Higgs couplings to intermediate gauge bosons scale quadratically with the W, Z masses,

$$g_V = 2\,\frac{M_V^2}{v} \quad (16)$$

These relations of the couplings g_f, g_V to the corresponding masses are a consequence of the electroweak symmetries and thus serve as a test of these symmetries. In order to unify the symmetry tests of the Higgs couplings to fermions and W, Z bosons the following coupling factors can be defined generically

$$\lambda_f = \kappa_f \frac{g_f}{\sqrt{2}} \quad (17)$$

$$\lambda_V = \sqrt{\kappa_V \frac{g_V}{2v}} \quad (18)$$

In this way these coupling factors scale *linearly* with the corresponding mass, if the SM predictions are correct,

$$\lambda_f = \kappa_f\, \frac{m_f}{v} \quad (19)$$

$$\lambda_V = \sqrt{\kappa_V}\, \frac{M_V}{v} \quad (20)$$

These relations are valid in any scheme for the masses and at any scale choice in general. If shown in a single plot showing λ_f and λ_V as a function of the corresponding fermion and gauge boson masses the electroweak symmetries can be tested by comparing the coupling factors with the line predicted by the SM for all coupling factors. Any deviation of the scaling factors κ_i from unity would move the data points off the line predicted by the SM. This method goes back to the old work of ACFA for a future e^+e^- collider [6] as shown in Fig. 1 which shows the expected accuracies at a future linear e^+e^- colliders with energies up to 700 GeV.

References

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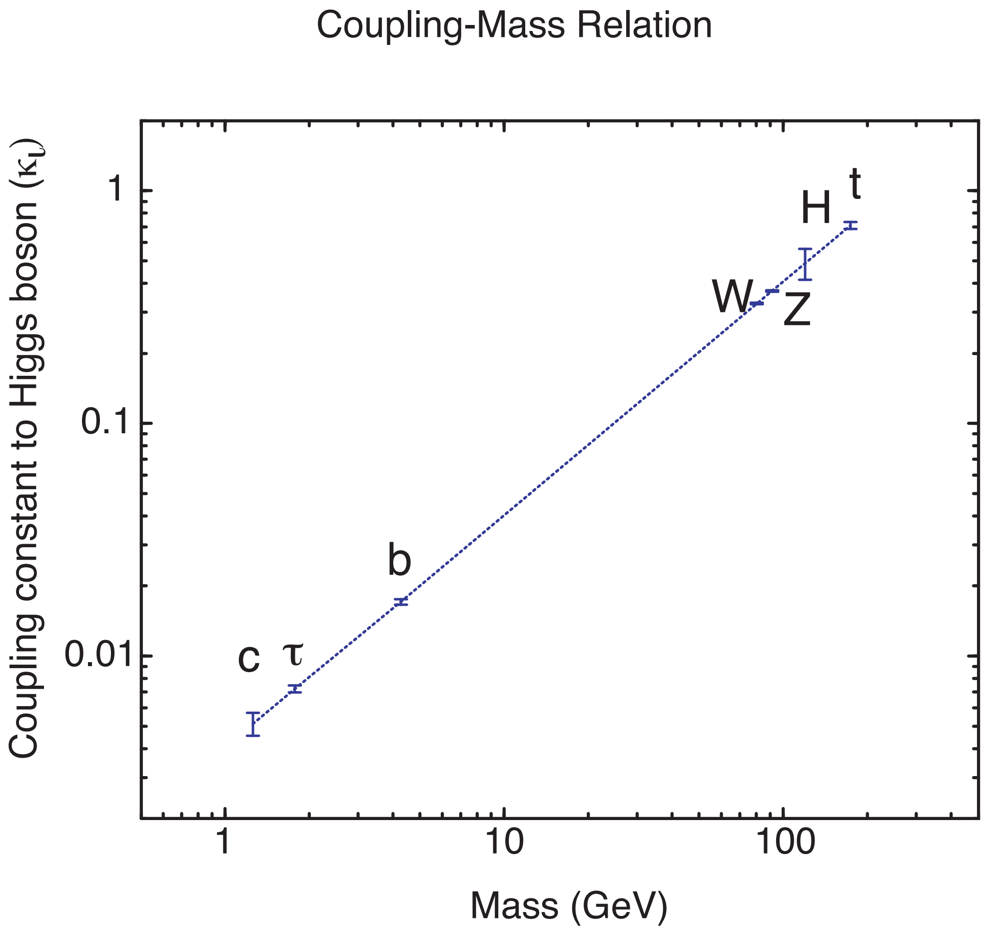

Figure 1: *The relation between the Higgs coupling of a particle and its mass in the SM. The error bars correspond to the accuracy expected from the ILC combining provisional data at 300, 500 and 700 GeV c.m. energies; Ref. [6].*

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