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A Probabilistic Data Association Filter for fast tracking in the ATLAS Transition Radiation Tracker

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Abstract

The ATLAS Level 2 Trigger algorithm for simultaneous pattern recognition and track fitting in the ATLAS Transition Radiation Tracker (TRT) is presented. The algorithm employs the Probabilistic Data Association Filter (PDAF) as a tool for "hit-to-track" association and track parameter estimation. An introduction to the PDAF technique and details of its implementation for tracking and electron identification in the TRT are given. The results of tests on simulated data with high track multiplicity are presented and discussed.
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1. Introduction

The Transition Radiation Tracker (TRT) is a combined tracking and electron identification detector that, together with silicon-strip (SCT) and pixel detectors comprises the inner detector of the ATLAS experiment [1]. The TRT consists of 370,000 cylindrical drift tubes—straws (4 mm diameter) arranged into cylindrical (in the TRT barrel) or wheel-shaped (in the TRT endcaps) layers (Fig. 1). The layers of straws are interleaved with polypropylene radiators which produce transition radiation (TR) when electrons traverse them. The TR is absorbed inside the straws producing hits with high energy deposits (TR hits) allowing the identification of electrons. The TR hits are separated from non-TR ones by the TRT readout electronics using a two-threshold discriminator.

The TRT provides the tracking information to the Level 2 (LVL2) Trigger system. One of the possible strategies for the LVL2 track reconstruction in the TRT is to combine stand-alone track finding in the Pixel and SCT [2] with subsequent propagation of the found tracks into the TRT using simultaneous "hit-to-track" association and track fitting. In the following this approach is referred to as track following. Due to the limited time budget of the LVL2 trigger which is 10 ms per region of interest on an 8 GHz CPU, a fast track following algorithm is required. The algorithm has to be sufficiently robust to cope with the high occupancy in the TRT (15–50%) and also able to resolve intrinsic ambiguities of the straw hits. To meet all these requirements, the LVL2 trigger employs the Probabilistic Data Association Filter (PDAF) [3].

2. The PDAF algorithm

The PDAF algorithm has been originally developed for tracking in radar systems. The basic assumption of the algorithm is that measurements (hits) are arranged into groups (e.g. layers in the TRT) such that the track of interest (ToI) can produce *at most* one hit per group. All the other hits in a group are assumed to be due to background tracks.

Let a track **T** be extrapolated to the kth TRT layer with $k = 1, 2, \ldots$ (Fig. 2). The PDAF selects only hits falling into the validation gate (VG)—a region set up around the extrapolated track position at a layer. Due to background,

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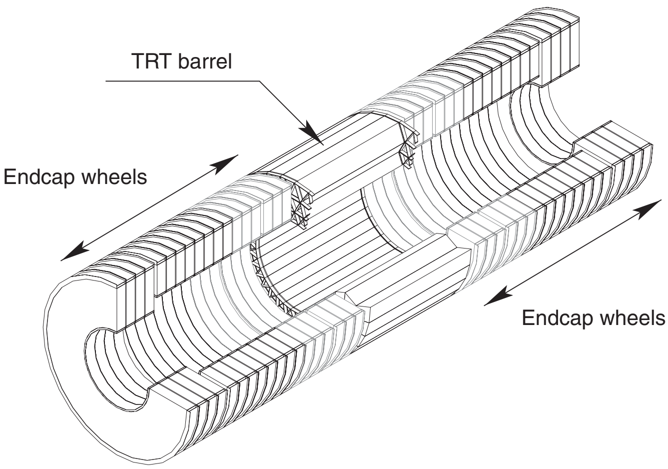

Fig. 1. The ATLAS Transition Radiation Tracker.

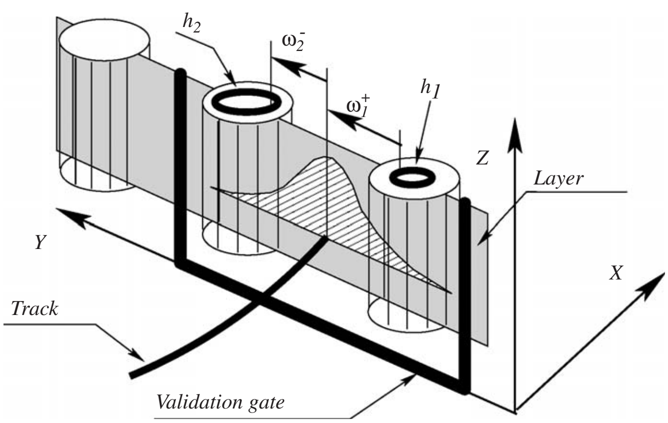

Fig. 2. The track following in the TRT.

the number of the validated hits N_k can be larger than one. The key element of the PDAF is the probabilistic data association (PDA) procedure that constructs and evaluates "hit-to-track" association hypotheses explaining the origin of the validated hits. For example, the validated hits h_1 and h_2 in Fig. 2 can be explained by one of the hypotheses from Table 1. The hypothesis H_i^+ with $i = 1, \ldots, N_k$ postulates that the ith hit is produced by the ToI **T**, the other $N_k - 1$ hits are due to background, and the correct sign of the drift distance ρ_i is "+" . The "no-detection" hypothesis H_0 means that the track is not detected and all hits are caused by background.

The posterior probabilities of the hypotheses $P(\mathrm{H}_0) = P_0$ and $P(\mathrm{H}_i^{\pm}) = P_i^{\pm}$, with $i = 1, \ldots, N_k$ are evaluated using Bayes rule:

$$P_i \propto p(\{\rho\}|A_i)\pi_i \tag{1}$$

where $\{\rho\}$ is the set of drift distance measurements for the validated hits, A_i is the "hit-to-track" association postulated by the hypothesis, $p(\cdot|\cdot)$ is a joint p.d.f. of the measurements $\{\rho\}$ conditioned upon the A_i, and π_i is a prior probability of the A_i. For statistically independent hits, the joint p.d.f. is the product of their p.d.f.s:

Table 1
An example of hit association hypotheses

Hits	H_0	H_1^+	H_1^-	H_2^+	H_2^-
h_1	**BG**	**T+**	**T−**	**BG**	**BG**
h_2	**BG**	**BG**	**BG**	**T+**	**T−**

$$p(\{\rho\}|A_i) = \prod_{j=1}^{N_k} \begin{cases} p_{\mathrm{BG}}(\rho_j), & j \neq i, \\ p_{\mathrm{T}}(\rho_j), & j = i \end{cases} \tag{2}$$

where $p_{\mathrm{T}}(\cdot)$ and $p_{\mathrm{BG}}(\cdot)$ are the p.d.f.s of the true and background-induced hits:

$$p_{\mathrm{BG}} = \frac{1}{R_{\mathrm{S}}}, \quad p_{\mathrm{T}}(\rho) = \frac{1}{\sqrt{2\pi S_k}} \exp\left(-\frac{\omega^2}{2S_k}\right) \tag{3}$$

where R_{S} is a straw radius, ω is the "track-hit" residual signed according to the association A_i, and S_k is the residual covariance.

The prior probability π_i is calculated assuming that every straw in a layer "generates" background hits with a probability P_{BG} which depends on the average occupancy of a layer. This assumption is valid because the TRT straws are arranged in layers in such a way that all straws in a layer have the same average occupancy. If the straw contains a background hit it becomes insensitive to the ToI. Under these assumptions, the prior probability of the associations A_i^+ and A_i^-, with $i = 1, \ldots, N_k$ is

$$\pi_i = (1 - P_{\mathrm{BG}})^{M_k+1} P_{\mathrm{BG}}^{N_k-1} \varepsilon P_D^{(i)} \tag{4}$$

where M_k is the number of straws without hits in the VG ("empty" straws), ε_{s} is the hit efficiency and $P_D^{(i)}$ is the probability of the track passing through the ith straw. If the extrapolated track position estimate is a Gaussian (Fig. 2) with mean $\widetilde{y}_k$ and variance γ_k, the probability $P_D^{(i)}$ is

$$P_D^{(i)} = \frac{1}{\sqrt{2\pi\gamma_k}} \int_{y_i - R_{\mathrm{S}}}^{y_i + R_{\mathrm{S}}} \exp\left(-\frac{(y - \widetilde{y}_k)^2}{2\gamma_k}\right) \mathrm{d}y. \tag{5}$$

The prior probability of the "no-detection" association A_0 (all hits are due to the background) reads

$$\pi_0 = (1 - P_{\mathrm{BG}})^{M_k} P_{\mathrm{BG}}^{N_k} (1 - \varepsilon P_D^{\mathrm{E}}) \tag{6}$$

where P_D^{E} is the total probability of the track passing through the "empty" straws.

Inserting Eqs. (4), (6), and (2) into Eq. (1) gives the equations for the association probabilities P_0, $P_i^{\pm}$, with $i = 1, \ldots, N_k$. After normalization, we get

$$P_0 = \frac{1}{1 + C\sum_{j=1}^{N_k} P_D^{(j)}(p_{\mathrm{T}}(+\rho_j) + p_{\mathrm{T}}(-\rho_j))}, \tag{7}$$

$$P_i^{\pm} = \frac{CP_D^{(i)} p_{\mathrm{T}}(\pm\rho_i)}{1 + C\sum_{j=1}^{N_k} P_D^{(j)}(p_{\mathrm{T}}(+\rho_j) + p_{\mathrm{T}}(-\rho_j))} \tag{8}$$

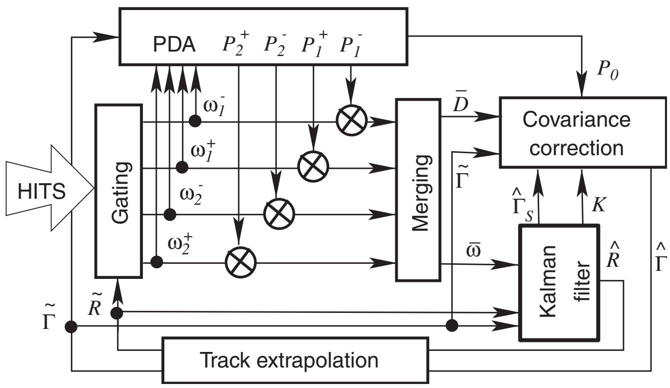

Fig. 3. The flow chart of the PDAF-based tracking.

where

$$C = \frac{\varepsilon R_{\mathrm{S}}(1-P_{\mathrm{BG}})}{P_{\mathrm{BG}}(1-\varepsilon P_D^{\mathrm{E}})}.$$

The probabilities given by Eqs. (7), (8) together with the hit residuals $\omega_i^{\pm}$, with $i = 1, \ldots, N_k$ are used in the track updating part of the PDAF algorithm (Fig. 3). As it can be seen, the residuals are weighted by the association probabilities and then merged into a single residual $\bar{\omega}$:

$$\bar{\omega} = \sum_{i=1}^{N_k} (P_i^+ \omega_i^+ + P_i^- \omega_i^-). \tag{9}$$

The update of the track parameters estimate $\widehat{X}_k$ is done by an embedded Kalman filter (KF) [4] in accordance with the following formula:

$$\widehat{X}_k = \widetilde{X}_k + K_k \bar{\omega} \tag{10}$$

where $\widetilde{X}_k$ is the extrapolated estimate, K_k is the standard Kalman gain.

The covariance $\widehat{\Gamma}_k$ of the estimate (10) is calculated by the covariance correction block (Fig. 3):

$$\widehat{\Gamma}_k = P_0 \widetilde{\Gamma}_k + (1-P_0)\widehat{\Gamma}_{\mathrm{s}} + K_k D K_k^{\mathrm{T}}, \tag{11}$$

where $\widetilde{\Gamma}_k$ is the covariance of the $\widetilde{X}_k$, $\widehat{\Gamma}_{\mathrm{s}}$ is the covariance produced by the KF, and D is the empirical dispersion of the residuals:

$$D = \sum_{i=1}^{N_k} (P_i^+(\omega_i^+)^2 + P_i^-(\omega_i^-)^2) - (\bar{\omega})^2. \tag{12}$$

The estimate Eq. (10) and covariance Eq. (11) are extrapolated to the next TRT layer where the PDAF cycle is repeated.

In order to identify electrons the ratio of the TR hits N_{TR} and the total number of TRT hits N_{T} associated with a track is calculated. With the PDAF-based tracking, this ratio is *estimated* from the association probabilities Eqs. (7) and (8):

$$\frac{N_{\mathrm{TR}}}{N_{\mathrm{T}}} = \frac{\sum_{k=1}^{K}\sum_{i\in\{\mathrm{TR}\}}^{N_k}(P_i^+ + P_i^-)}{K - \sum_{k=1}^{K} P_0(k)} \tag{13}$$

where K is the total number of the TRT layers crossed by the track and {TR} denotes the set of all validated TR hits.

It should be mentioned that recently a few algorithms similar to the PDAF have been proposed for the tracking in particle physics experiments. One of them is the Gaussian-sum filter (GSF) [5] which is equivalent to the PDAF if all Gaussian components (track parameter estimates) are merged into a single component after each TRT layer. However, such a truncated GSF is 2–4 times slower than the PDAF. For the example shown in Fig. 2, the GSF runs four (2 hits $\times$ 2 possible drift distance signs) KFs in parallel and then merges the filter outputs into a single estimate. On the other hand, the PDAF first merges all four residuals into one and then uses a single KF with the covariance correction to obtain the updated estimate of track parameters.

Another PDAF-like algorithm is the Deterministic Annealing Filter (DAF) [6]. The DAF iteratively refines the hit association probabilities through passes of re-weighted forward/backward filtering with gradually lowered measurement variance (annealing). In contrast, the PDAF is a fast algorithm which does not foresee iterations and annealing and therefore is more suitable for the LVL2 trigger application. In addition, the PDAF employs a more elaborated hit association method which explicitly allows for the "no-detection" hypothesis.

3. Algorithm performance evaluation

The performance of the PDAF algorithm has been studied using the full ATLAS Monte Carlo (MC) simulation. The signal events were simulated by electrons with p_{T} of 20 GeV—the lowest p_{T}-threshold expected to be used in the LVL2 trigger for discovery (e.g. Higgs) physics. Background events were simulated by superimposing Poisson-distributed (with average λ equal to 0 (signal only), 4.5 and 23) inelastic *pp* collisions.

The following criteria have been used to assess the algorithm performance

- *hit association quality*—average association probabilities $\langle P_i \rangle$ for the true and background hits;
- *robustness*—dependence of the hit association quality on the occupancy.

The performance has been evaluated independently for two regions of the TRT—barrel (BR) ($|\eta| \leqslant 0.7$) and endcaps ($1.1 \leqslant |\eta| \leqslant 2.5$). For both regions, the VG width was set to 8 mm.

The results obtained are summarized in Table 2. As it can be seen, the PDAF identifies true hits with 93–97% efficiency and rejects half of the background hits. The stability of the estimated $N_{\mathrm{TR}}/N_{\mathrm{TRT}}$ ratio across the whole occupancy range shows the robustness of the PDAF. The hit association quality drops at high η due to the large amount of material traversed by a track before the

Table 2
The hit association performance

Dataset	$\lambda = 0$		$\lambda = 4.5$		$\lambda = 23$	
TRT region	BR	EC	BR	EC	BR	EC
Occupancy	0.01	0.03	0.17	0.16	0.6	0.5
$\langle P_i \rangle$, true hits	0.98	0.93	0.97	0.93	0.95	0.93
$\langle P_i \rangle$, background	0.52	0.52	0.50	0.49	0.49	0.49
Ratio N_{TR}/N_{TRT}	0.23	0.23	0.23	0.24	0.25	0.25

Table 3
The average total and splitted CPU times

PDAF total time, ms	2.5
PDA procedure	0.65
Track update	0.45
Track extrapolation	1.1
KF total time, ms	1.5

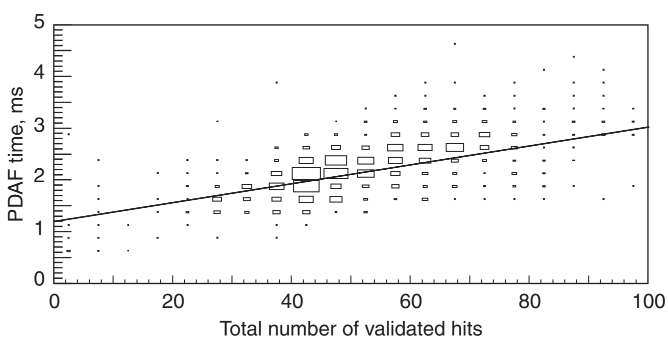

Fig. 4. The PDAF CPU time vs. the total number of validated hits.

TRT—the total radiation length before the TRT is $X_0 = 0.2$ for $\eta = 0$ and $X_0 = 0.6$ for $\eta = 2.0$.

The CPU time of the algorithm has been measured on a Xeon 2.4 GHz processor as a function of the total number of the validated hits N_{val} along a track. The dataset with the highest occupancy ($\lambda = 23$) was used. As a reference, the timing of the standard KF has been also measured. Table 3 shows the average total (T) time of the PDAF and KF and the time of the main PDAF blocks.

Table 4
Straight-line fit parameters of the $T(N_{val})$ dependence

Algorithm	Offset, ms	Slope
PDAF	1.2	0.018
KF	1.1	0.01

The $T(N_{val})$ distribution for the PDAF is shown in Fig. 4. The parameters of a straight line fitted to the $T(N_{val})$ data are given in Table 4 for both PDAF and KF. As it can be seen, the computational requirements of the PDAF are comparable with those of the KF. According to Table 3 two main blocks of the PDAF algorithm, the PDA procedure and track update, take, respectively, 30% and 20% of the total CPU time.

4. Conclusion and outlook

A new algorithm based on the Probabilistic Data Association Filter (PDAF) has been developed for track reconstruction in the ATLAS TRT. The tests on simulated data with high track multiplicity have shown that the PDAF is a computationally efficient and robust algorithm well suited for the planned trigger application.

The algorithm has been integrated into the ATLAS Level 2 Trigger software. The evaluation of the algorithm performance in the ATLAS Level 2 Trigger framework is ongoing.

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