

Error calculation for weighted unbinned ML fits (in RooFit)

Max Baak, CERN
Wouter Verkerke, Nikhef

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Problem of event weights

Use cases:

- Combination of MC samples with different luminosities
- MC@NLO events: positive and negative event weights

When using event weights in unbinned maximum likelihood fit:

- Minimum found is correct (usual caveats)
- **Associated errors are incorrect**
 - "Statistics mismatch".
 - Associated errors scale with sum of the weights.
 - Should scale with actual statistics of MC sample.

Example:

- For uniform weights of $w=2$, errors underestimated by $\sqrt{2}$
 - But, information content has not changed, so errors should not change.

HESSE covariance matrix

- Given MC sample with event weights.
 - When $-\log(L)$ is passed to Minuit ...
 - ... it return uncorrected covariance matrix COV.
- Errors scale with:
1/sum of event weights
- (incorrect errors!)
- MC errors representative for errors expected on data.
 - $N(\text{data}) = \text{sum weights}$

$\vec{\alpha}$ \equiv Fit parameters

\vec{x}_i \equiv Observables for event i

$\text{pdf}(\vec{x}_i, \vec{\alpha})$ \equiv p_i

w_i \equiv Event weight for event i

$$-\log L(\vec{\alpha}) = -\sum_{i=1}^N w_i \log[\text{pdf}(\vec{x}_i, \vec{\alpha})]$$

$$G_{\mu\nu} = \sum_{i=1}^N w_i \frac{dp_i}{d\alpha_\mu} \frac{dp_i}{d\alpha_\nu}$$

$$\begin{aligned} \text{COV}(\vec{\alpha}, \vec{\alpha}) &= G^{-1} \\ &\equiv \text{HESSE covariance matrix} \end{aligned}$$

Corrected covariance matrix

- Obtain event-weight corrected covariance matrix by recomputing covariance matrix. (I'll skip the derivation)
 - Note: parabolic approximation of $-\log(L)$ (ala HESSE)

$$G_{\mu\nu} = \sum_{i=1}^N w_i \frac{dp_i}{d\alpha_\mu} \frac{dp_i}{d\alpha_\nu}$$

$$F_{\mu\nu} = \sum_{i=1}^N (w_i)^2 \frac{dp_i}{d\alpha_\mu} \frac{dp_i}{d\alpha_\nu}$$

$$\text{COV}(\vec{\alpha}, \vec{\alpha}) = G^{-1} F G^{-1}$$

- Errors scale with: $\text{sum weight}^2 / \text{sum weight}$
 - "Statistics of MC sample"

Triviality requirements: $w_i = 1$: $\text{COV}(\vec{\alpha}, \vec{\alpha}) = G^{-1}$

$w_i = \text{constant}$: $\text{COV}(\vec{\alpha}, \vec{\alpha}) = \text{COV}(\vec{\alpha}, \vec{\alpha})[w_i = 1]$

Fit to Gaussians

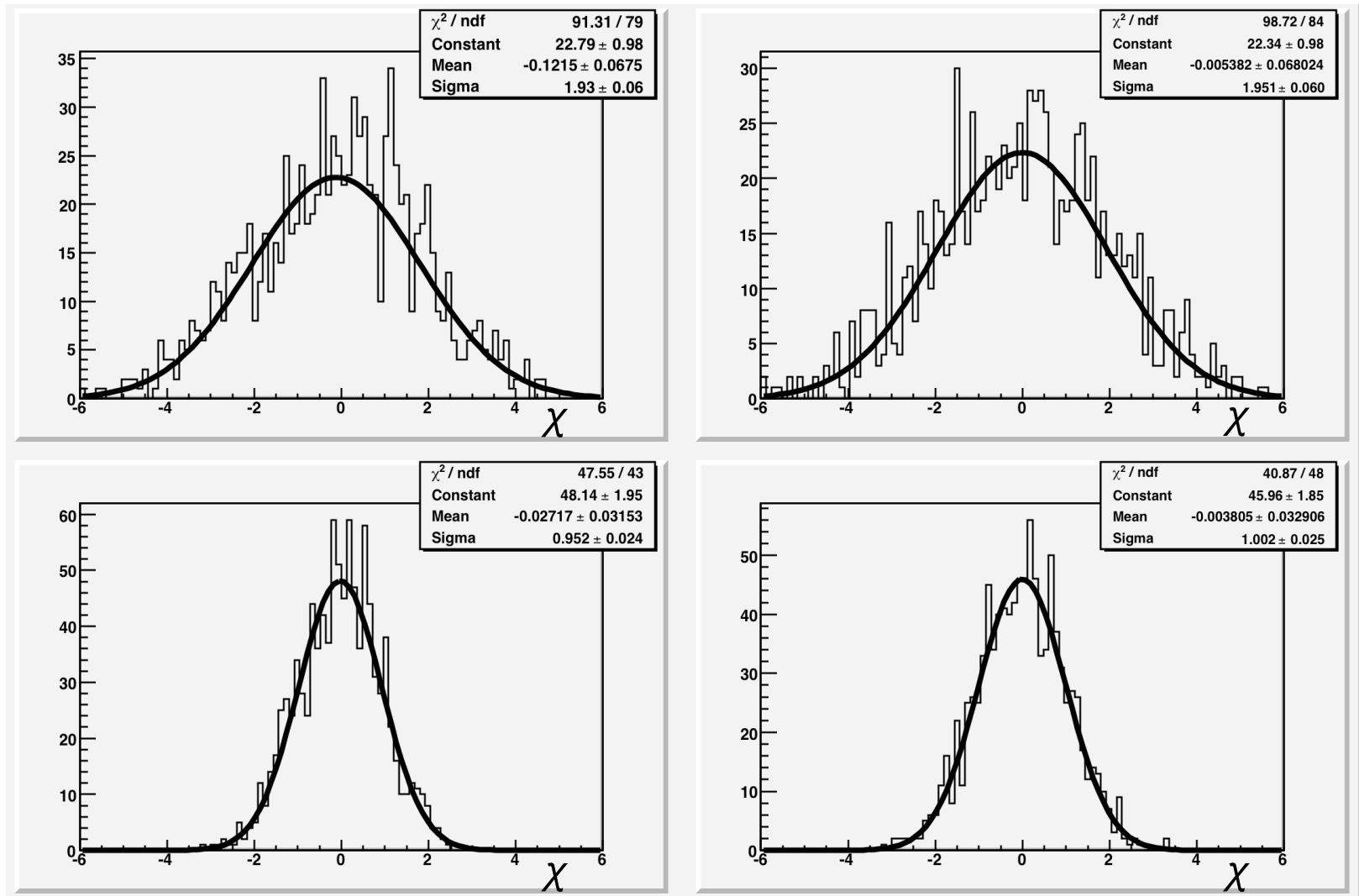
- Fit mean and width of simple Gaussian toy MC.
- Event weights, generated uniformly in [-2,4]

$$\chi = \frac{a_{fit} - a_{gen}}{\sigma(a)}$$

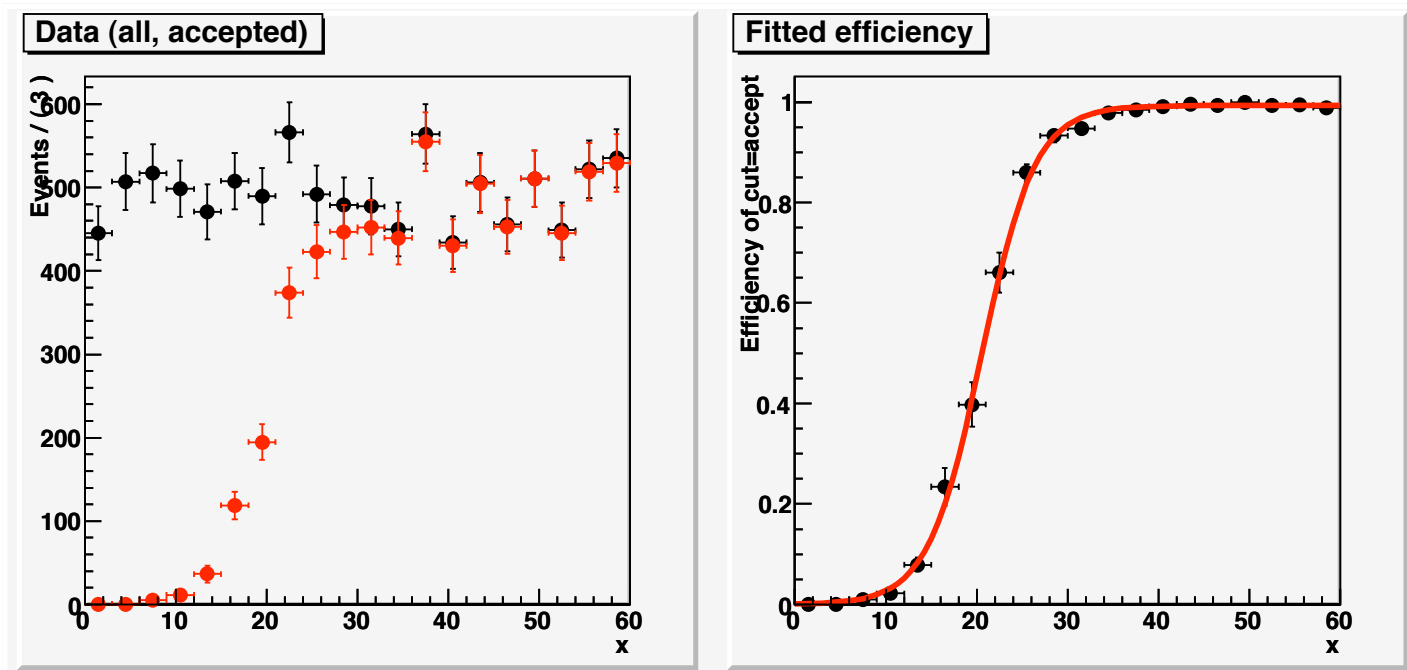
Top: pull distributions with uncorrected fit parameter errors.

Bottom: pull distributions with corrected fit parameter errors.

Bottom distributions consistent with zero mean and unit width.

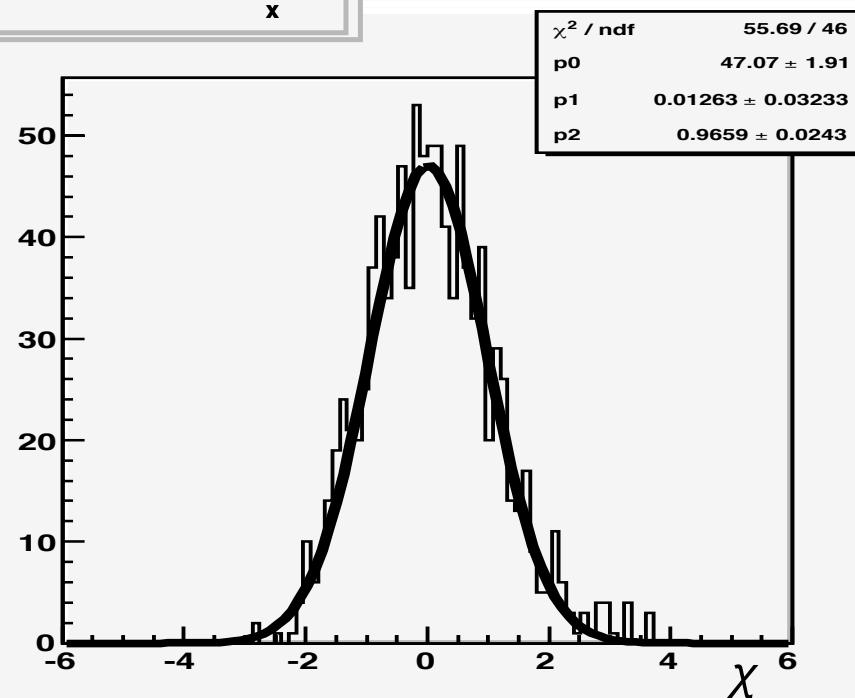


Efficiency curve fit



- Unbinned efficiency curve fit
- Event weights, uniform in [-1,3]

- All pull distributions of fit parameters are uniform Gaussians. (see backup)
- Example: pull distribution of 'plateau' fit parameter.
 - Plateau at 0.99



MC@NLO ttbar sample

- MC@NLO semi-leptonic ttbar sample
- Plot shows reconstructed hadronic top mass
- # MC events = 858
- Sum of weights = 650

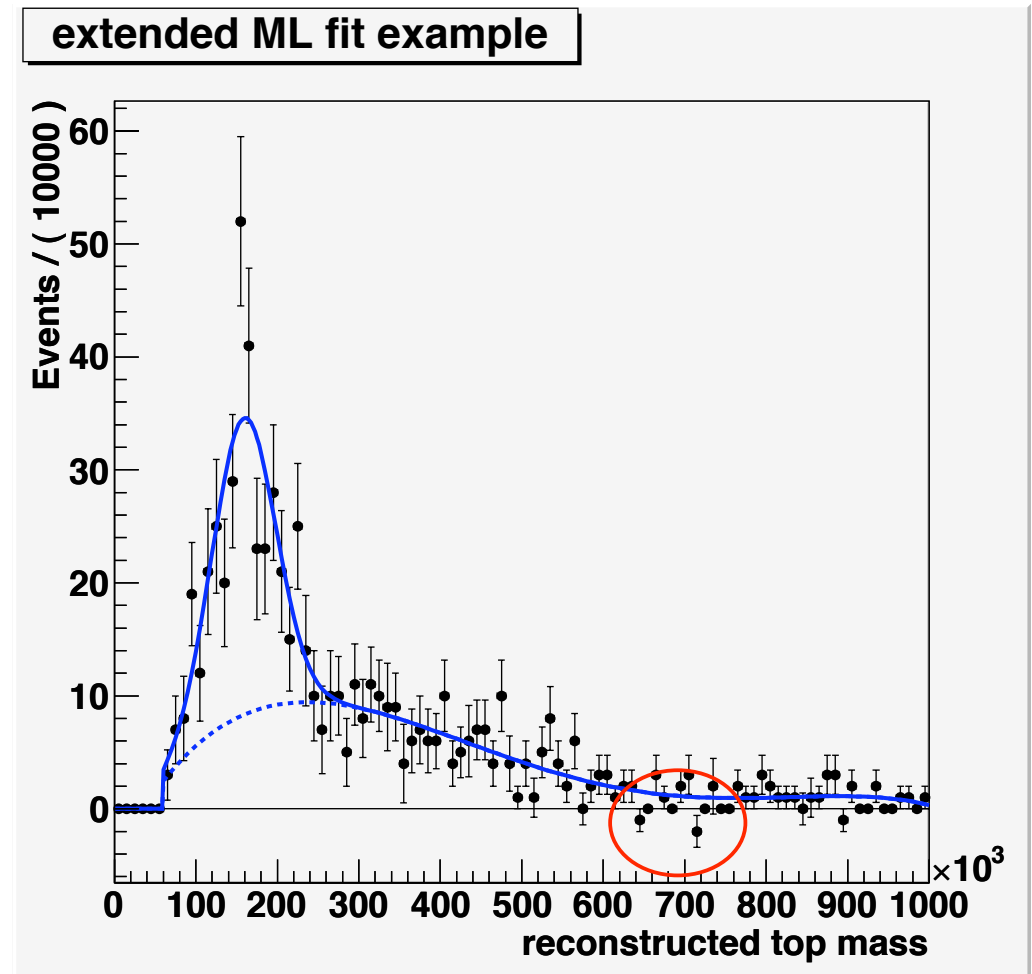
- **Uncorrected fit values:**

Parameter	Value	Error
nbkg	3.95875e+02	3.78459e+01
nsig	2.54127e+02	3.59238e+01

- **Corrected MC fit errors:**

nbkg	3.95875e+02	5.50498e+01
nsig	2.54127e+02	5.33091e+01

- **Note: significant underestimate of original fit errors!**



Usage of weights in RooFit

- ROOT v5.23.04 (latest development version)
 - HESSE cov. matrix determined twice (with w_i and w_i^2), then multiplied.

```
// set weight observable
dataset->setWeightVar(weightvar) ;

// default option: errors from original HESSE error matrix
// errors are proportional to the sum of the weights ("errors expected on data")
model.fitTo(*data,SumW2Error(kFALSE)) ;

// sum-of-weights corrected HESSE error matrix
// errors are proportional to true MC statistics
model.fitTo(*data,SumW2Error(kTRUE)) ;

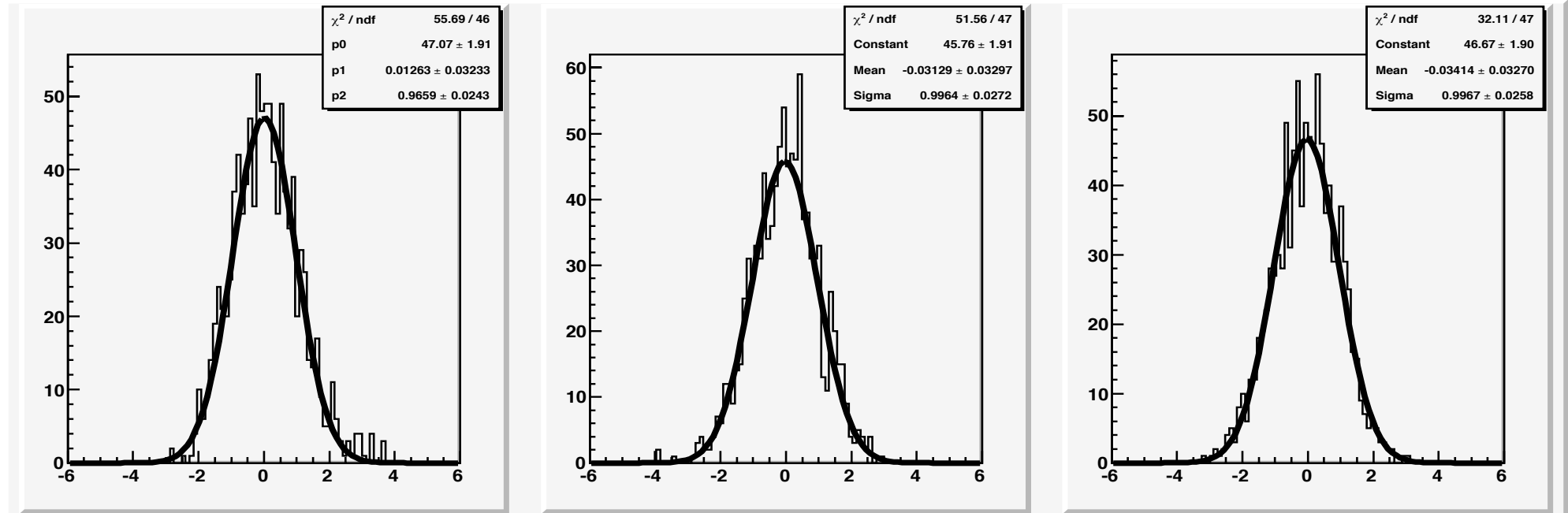
// plot weighted events
data->plotOn(frame,DataError(RooAbsData::SumW2)) ;
```


Conclusion

- RooFit can now properly calculate errors for weighted unbinned maximum likelihood fits
 - ROOT v5.23.04 or greater
 - /afs/cern.ch/sw/lcg/app/releases/ROOT/5.23.04/slc4_amd64_gcc34
 - For example, proper statistical treatment of negative weights.
- Works for HESSE or MIGRAD error calculation (not MINOS)
- Intend to write down corrected covariance matrix calculation in brief Atlas statistics note.
- Find my examples at:
 - <https://twiki.cern.ch/twiki/bin/view/Main/RooFit>
- References:
 - Eadie et al. (Section 8.5, p. 172 ff)
 - Babar internal note #656

Backup slide

- Turn-on curve, plateau at 0.99



- Plateau at 0.80

