Background-only hypothesis test with different methods for calculating significance

Adi Ashkenazi (CERN)

Renaud Bruneliere (Freiburg)

Riccardo Bianchi (Freiburg)

Outline

- Methods for calculating significance
- Results
- Implementation example
- Conclusions

Methods

- Poisson distribution from data in signal region $$\frac{e^{-\mu}\mu^{n}}{n!}$$

Methods

- Poisson distribution from data in signal region $\frac{e^{-\mu}\mu^{n}}{n!}$

- Due to uncertainty in $\mu = \mu_s + \mu_b$:
  multiply the Poisson with a Poisson/Gaussian from extra measurement

$$\frac{e^{-\mu_s+\mu_b}\left(\mu_s+\mu_b\right)^n}{n!}\times\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{\left(\mu_b-\langle\mu_b\rangle\right)^2}{2\sigma^2}} \qquad \frac{e^{-\mu_s+\mu_b}\left(\mu_s+\mu_b\right)^n}{n!}\times\frac{e^{-\tau\mu_b}\left(\tau\mu_b\right)^{n_{off}}}{n_{off}!}$$

μ_s is signal, μ_b is background = nuisance parameter

Methods

- Poisson distribution from data in signal region

$$\frac{e^{-\mu}\mu^{n}}{n!}$$

- Due to uncertainty in μ:

  multiply the Poisson with a Poisson/Gaussian from extra measurement

$$\frac{e^{-\mu_s+\mu_b}\left(\mu_s+\mu_b\right)^n}{n!}\times\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{\left(\mu_b-\langle\mu_b\rangle\right)^2}{2\sigma^2}} \qquad \frac{e^{-\mu_s+\mu_b}\left(\mu_s+\mu_b\right)^n}{n!}\times\frac{e^{-\tau\mu_b}\left(\tau\mu_b\right)^{n_{off}}}{n_{off}!}$$

μ_s is signal, μ_b is background = nuisance parameter

- In order to get rid of nuisance parameters, can either integrate or minimize over them
- Statistical significance for background only hypothesis = pvalue = sum from nobs to infinity

Methods

- Z_{Bi} Binomial : exact classical solution

Methods

- Z_{Bi} Binomial : exact classical solution

- Hybrid recipe with Integration:

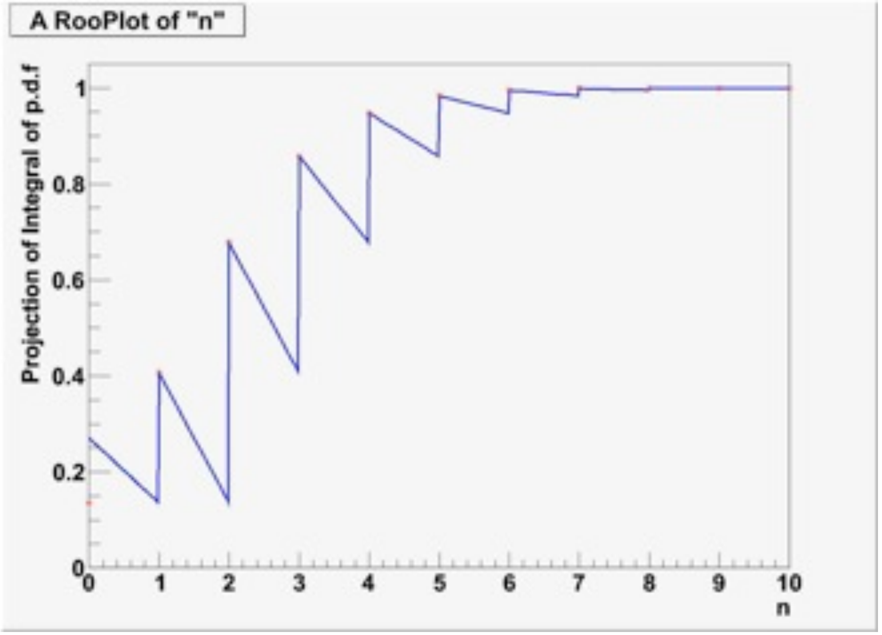
Z_{Γ}

Z_N

- Using Cumulative Distribution Function
- Using CreateIntegral

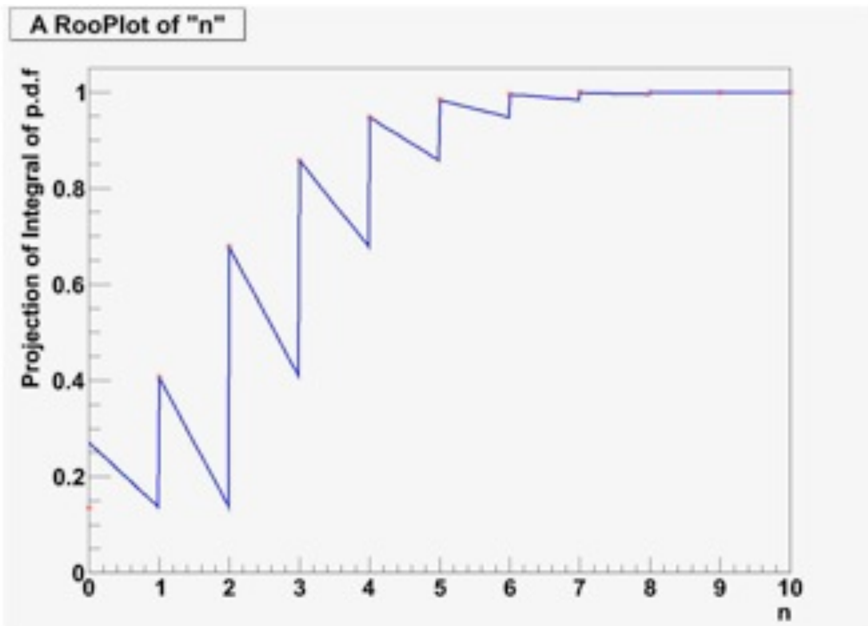
Problem with c.d.f

Before

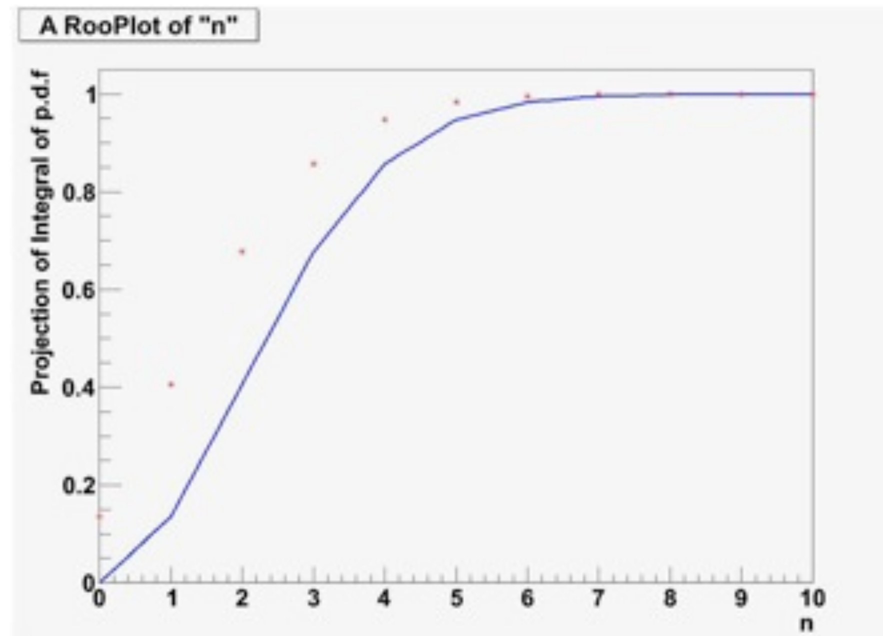


Problem with c.d.f

Before

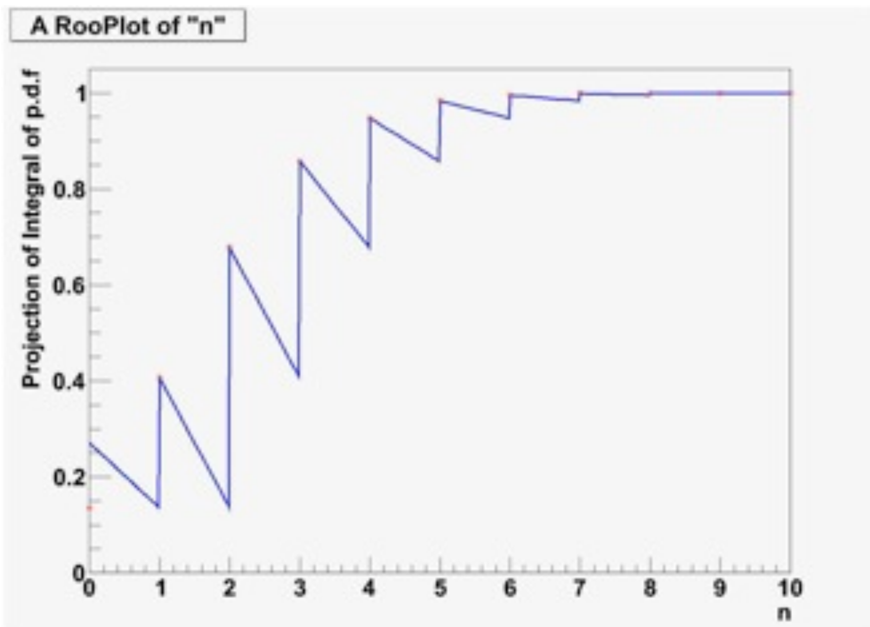


After

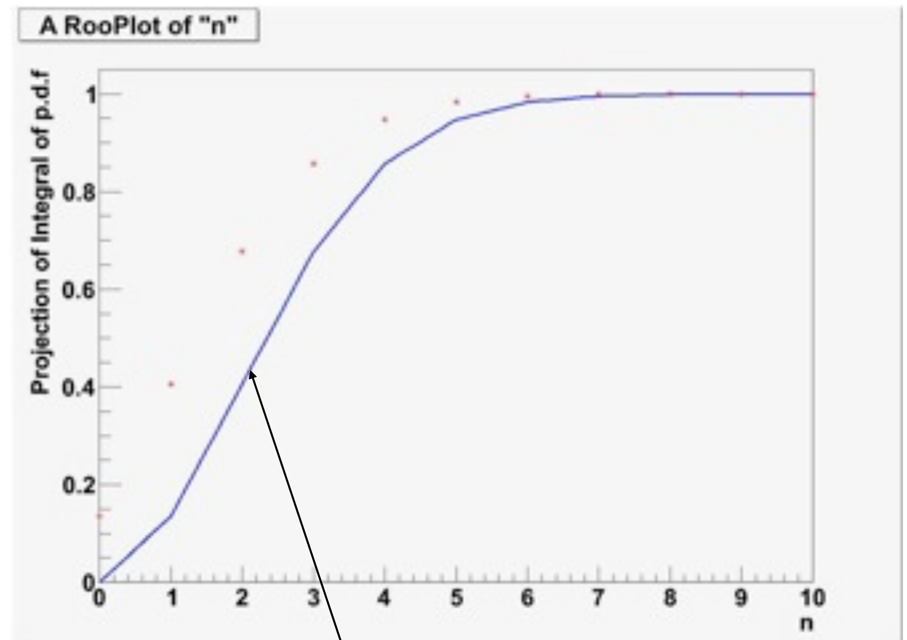


Problem with c.d.f

Before



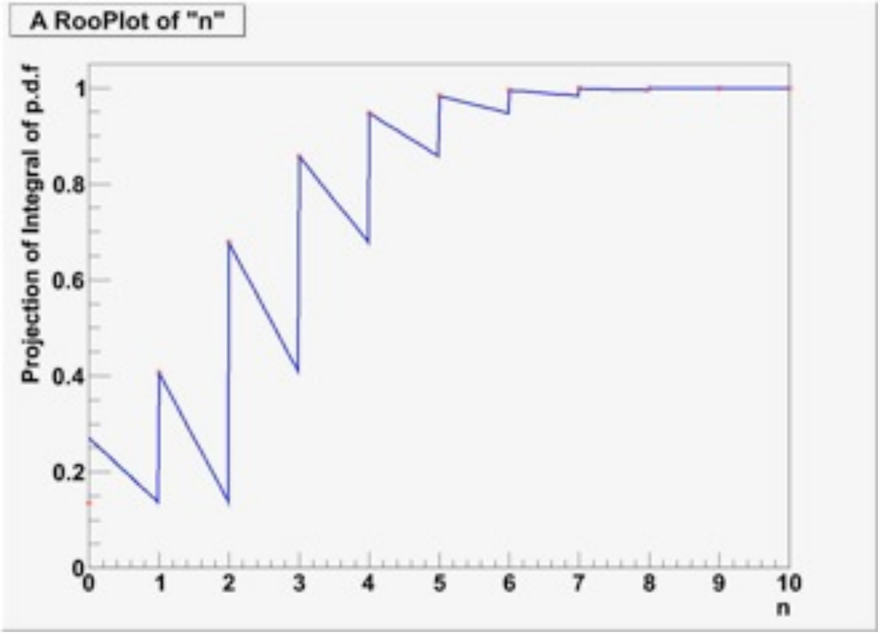
After



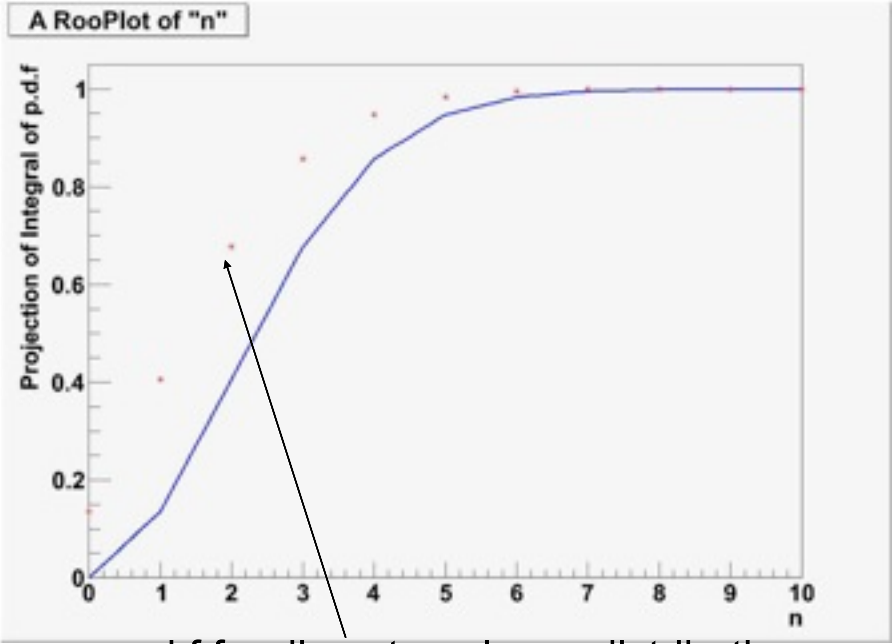
- c.d.f for as if continuous poisson distribution = ∫ f(x)dx
- pvalue(n) = 1 – c.d.f(n)
- what is in RooStats now

Problem with c.d.f

Before



After



- c.d.f for discrete poisson distribution $= \Sigma f(n)$
- pvalue(n) = 1 – c.d.f(n - 1)

Methods

- Z_{Bi} Binomial : exact frequentist solution

- Hybrid recipe with Integration:

  Z_{Γ}

  Z_N

  - Using CDF
  - Using CreateIntegral

- Likelihood profile = minimization :

  $Z_{PL} Poisson$

  $Z_{PL} Gaussian$

  - Using ProfileLikelihoodCalculator & MINUIT

Results

Significance Comparison										
n_{obs}	4.0		6.0		9.0		17.0		50.0	
μ_b	1.0		1.3		3.8		3.8		27.5	
σ_b	0.477		0.3		0.9		0.6		3.71	
Z_b	1.66	1.665	2.63	2.631	1.82	1.818	4.46	4.457	2.93	2.933
Z_Γ	1.66	1.666	2.63	2.688	1.82	1.907	4.46	4.464	2.93	3.008
Z_N	1.88	1.873	2.71	2.713	1.94	1.938	4.55	4.551	3.08	3.078
Z_{PL} *Gaussian*	2.00	2.013	2.83	2.831	2.02	2.019	4.62	4.624	3.10	3.104
Z_{PL} *Poisson*	1.95	1.948	2.82	2.816	1.99	1.990	4.57	4.574	3.02	3.023

Significance Comparison										
n_{obs}	67.0		200.0		523.0		498428.0		2119449.0	
μ_b	30.0		100.0		388.6		493434.0		2109732.0	
σ_b	7.75		31.6		8.1		702.4		433.8	
Z_b	2.89	2.893	2.20	2.203	5.93	5.921	5.01	5.012	6.40	6.405
Z_Γ	2.89	3.087	2.20	2.203	5.93	5.930	5.01	5.013	6.04	6.409
Z_N	3.44	3.429	2.90	2.900	5.93	5.948	5.02	5.018	6.40	6.409
Z_{PL} *Gaussian*	3.45	3.444	2.90	2.898	5.93	5.954	5.02	5.001	6.40	6.405
Z_{PL} *Poisson*	3.04	3.042	2.38	2.384	5.95	5.940	5.01	5.013	6.40	6.405

Results

Significance Comparison										
n_{obs}	4.0		6.0		9.0		17.0		50.0	
μ_b	1.0		1.3		3.8		3.8		27.5	
σ_b	0.477		0.3		0.9		0.6		3.71	
Z_b	1.66	1.665	2.63	2.631	1.82	1.818	4.46	4.457	2.93	2.933
Z_Γ	1.66	1.666	2.63	2.688	1.82	1.907	4.46	4.464	2.93	3.008
Z_N	1.88	1.873	2.71	2.713	1.94	1.938	4.55	4.551	3.08	3.078
Z_{PL} *Gaussian*	2.00	2.013	2.83	2.831	2.02	2.019	4.62	4.624	3.10	3.104
Z_{PL} *Poisson*	1.95	1.948	2.82	2.816	1.99	1.990	4.57	4.574	3.02	3.023

Paper results

Roostats results

Significance Comparison										
n_{obs}	67.0		200.0		523.0		498428.0		2119449.0	
μ_b	30.0		100.0		388.6		493434.0		2109732.0	
σ_b	7.75		31.6		8.1		702.4		433.8	
Z_b	2.89	2.893	2.20	2.203	5.93	5.921	5.01	5.012	6.40	6.405
Z_Γ	2.89	3.087	2.20	2.203	5.93	5.930	5.01	5.013	6.04	6.409
Z_N	3.44	3.429	2.90	2.900	5.93	5.948	5.02	5.018	6.40	6.409
Z_{PL} *Gaussian*	3.45	3.444	2.90	2.898	5.93	5.954	5.02	5.001	6.40	6.405
Z_{PL} *Poisson*	3.04	3.042	2.38	2.384	5.95	5.940	5.01	5.013	6.40	6.405

Very good agreement between Cousins et al. and our RooStats implementation

Conclusion

- Tested and implemented all methods described by Cousins et al. in Roostats

- Good agreement between their results and ours

- Simple code which could be used as short tutorial for Roostats and comparison between groups