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Some formulas for calculating cryogenic heat loads equivalent 4.5 K refrigerator load

Introduction

The total heat created in a SRF cryo-module occurs at various temperatures and is finally dissipated at ambient temperature – by means of a cryogenic installation (refrigerator, liquefier). The different heat loads are

1. The RF losses originating at the temperature of saturated liquid helium, normally at 4.5 K (1.3 bar) or at 2 K (0.03 bar);
2. The radiated heat onto the thermal shields at about 80 K;
3. The RF heat generated by in the walls of the coaxial input power line at 5 – 8 K, etc.

All these heat loads are created at a temperature below the ambient temperature. As due to the second law of thermodynamics no heat flows from a lower to a higher temperature, the heat must be transported through mechanical work by virtue of the first law of thermodynamics.

The aim of this paper is to calculate the mechanical work needed for the transport of heat from low to ambient temperature. Under these ideal conditions, treated in the first part (Useful relations), the entropy associated with the heat dissipated at low temperature is equal to the entropy associated with the heat liberated at ambient temperature; in other words: the entropy S is preserved for the transfer process, dS = 0. In a second part (Estimation of equivalent heat loads at 4.5 K), a more realistic computation of the heat load is presented[1].

Some typical quantities for helium are summarized in Tables 1 and 2.

Table 1: Characteristic numbers for Helium

Heat of evaporation	L	21	J/g
Density			
- gas at ambient temperature (300 K)	ρ	179	g/m^3
- liquid at ambient pressure (4.2 K)		125	g/l

Table 2: Several equivalences

1 m^3 gas @ 300 K	1.4 l liquid @ 4.2 K
1 Watt @ 4.2 K	1 m^3/h gas @ 300 K
21 Watt @ 4.2 K	1 g/s mass flow
4 Watt refrigeration @ 4.2 K	1l/h liquefaction
100 W refrigeration @ 4.2 K	1 g/s liquefaction

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[1] U. Wagner, private communication

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Useful relations[2]

Ideal cool down

$$\Delta W = \int_{Tc}^{Th} mc_p(T)\left(\frac{T_h}{T}-1\right)dT$$

Ideal liquefaction

$$\Delta W = \underbrace{\int_{Tc}^{Th} mc_p(T)\left(\frac{T_h}{T}-1\right)dT}_{\text{Work to extract sensible heat}} + \underbrace{mh_{fg}\left(\frac{T_h}{T_{nbp}}-1\right)}_{\text{Work to extract latent heat}}$$

$$\rightarrow \Delta e = \frac{\dot{W}}{\dot{m}} = T_h\Delta s - \Delta h \tag{1}$$

Example:

$$e = \frac{\dot{W}}{\dot{m}} = 300\text{ K}\cdot(31.5-3.3)\frac{\text{kJ}}{\text{kgK}} - (1554-10.5)\frac{\text{kJ}}{\text{kg}} = (8460-1544)\frac{\text{kJ}}{\text{kg}} = 6917\frac{\text{W}}{\text{g/s}}$$

Ideal refrigeration

$$\dot{W} = \dot{Q}_c\left(\frac{T_h}{T_c}-1\right) = \dot{m}\Delta h_L\left(\frac{T_h}{T_c}-1\right)$$

Example:

$$e = \frac{\dot{W}}{\dot{m}} = 21\,\frac{\text{W}}{\text{g/s}}\cdot\left(\frac{300}{4.2}-1\right) = 1480\frac{\text{W}}{\text{g/s}}$$

Ideal coefficient of performance (refrigerator)

$$COP = \frac{\dot{Q}_c}{\dot{W}} = \frac{1}{\left(\frac{T_h}{T_c}-1\right)} = \frac{T_c}{T_h-T_c}\,(<1),$$

from which results

$$\frac{1}{COP} = \frac{\dot{W}}{\dot{Q}} = \frac{\dot{W}/\dot{m}}{\dot{Q}/\dot{m}} = \frac{\Delta e}{\Delta h} = \frac{T_h\Delta s - \Delta h}{\Delta h}. \tag{2}$$

Figure of merit (% Carnot)

$$FOM_{\text{liquefier}} = \frac{(\dot{W}/\dot{m}_c)_{\text{ideal}}}{(\dot{W}/\dot{m}_c)_{\text{actual}}}\,(<1)$$

$$FOM_{\text{refrigerator}} = \frac{COP_{\text{actual}}}{COP_{\text{ideal}}}\,(<1)$$

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[2] Legend: h = hot, c = cold, fg = fluid-gas, nbp = normal boiling point), h_L = latent heat of evaporation

Estimation of equivalent heat loads at 4.5 K

In Table 3, the foregoing relations will be used to estimate, in a more realistic way, the heat loads of the SPL.

The cold box is considered as a collection of black boxes, each one consisting of an input and output of the fluid, characterized by its pressure p and temperature T, based on experience. The T-S diagram allows then determining the specific enthalpy h and the specific entropy s, at the input and output, respectively, as well as their differences. From eqs. 1 and 2 follows the COP.

Table 3: Determination of the COP

Purpose	T [K]	p [bar]		h [J/g] c.f. Annex	s [J/(gK)]	e [J/g] (eq. 1)	Δe [J/g]	Δh [J/g]	$\frac{1}{COP}$ (eq. 2)	$\frac{1}{COP}_{rel.\,4.5\,K}$
Thermal screening	50	18	gas	277	16.3	4613	558	132	4.2	0.059
	75	16	gas	409	18.6	5171				
Interception of heat for coupler etc.	5	3	gas	$h_1 =$ 14.6	4.2	$e_1 =$ 1245	$\Delta e = e_1 - e_3 =$ 2138	$\Delta h = h_1 - h_2 =$ 35	61	0.86
	8		gas	$h_2 =$ 49.5	9.9	$e_2 =$ 2921				
	7.25	1.3	gas	$h_3 =$ 49.5	11.4	$e_3 =$ 3383				
Load at 2 K	22.5	1.3	gas	132	17.6	5158	4060	$\Delta h = L$ = 21	193	2.7
	4.6	3	gas	12.3	3.7	1098				
Reference refrigerator load @ 4.5 K	4.5	1.3	liquid	11.2	4	1189	1154	16.3	71	1.0
			gas	27.5	7.9	2343				

In order to obtain the equivalent heat load for a refrigerator at 4.5 K, the actual heat load at a given temperature T must be multiplied by the factor $\frac{1}{COP}_{rel.\,4.5\,K}$, as listed in the last column of Table 3.
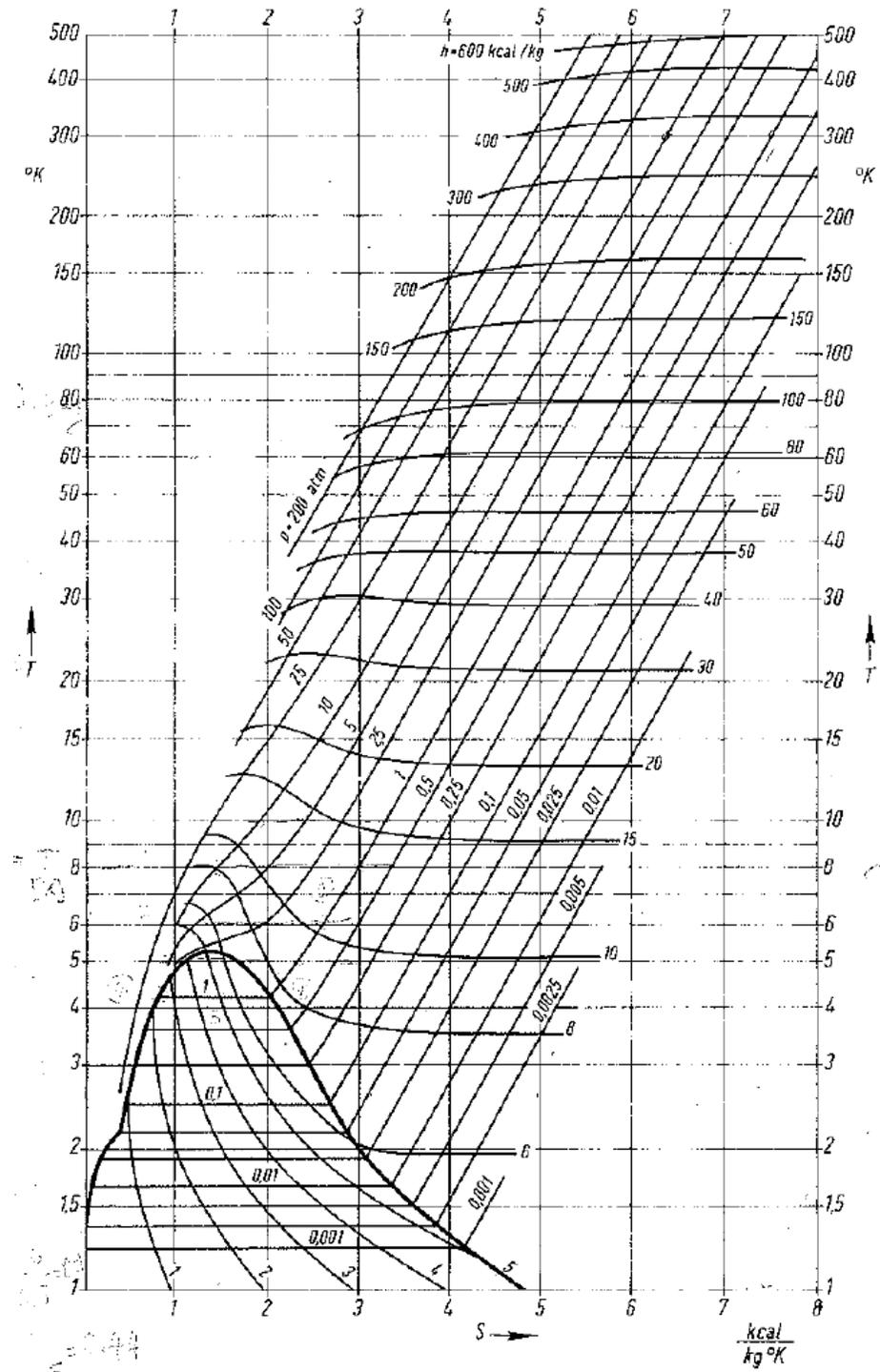
Annex: T-S diagram for Helium[3]

Abb. XII, 6. Temperatur-Entropie-Diagramm für Helium

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[3] 1 kcal = 4.187 kJ