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H-dibaryon

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Abstract

The quark cluster model studies concerning the H-dibaryon are reviewed. The covered topics are the H-dibaryon itself, the interaction between a nucleon and an H-dibaryon, and the one between two H-dibaryons. A related study on the H-dibaryon in nuclear matter is also reviewed and its implication to double hypernuclei is discussed.

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§1. Introduction

The H-dibaryon is a spin and isospin singlet, positive parity state composed of six quarks (uuddss). It was first proposed by Jaffe in 1977 using MIT bag model as a strongly bound state with its mass 81 MeV lighter than the $\Lambda\Lambda$ threshold [1]. Being the ground state in the $S = -2$ sector of a $B = 2$ system, the H-dibaryon is stable against the strong interaction and can decay only via the weak interaction. During two decades, the H-dibaryon has attracted much interest as a plausible candidate of exotic states, which are different from the hadrons known so far, i.e. mesons $q\bar{q}$ and baryons q^3.

Lots of efforts for H-dibaryon hunting have been made although there have been no conclusive experimental results on the existence of the H-dibaryon. [77)–129)] Many of the H-dibaryon search experiments have been made with nuclear reactions, in which the H-dibaryon is expected to be produced in nuclei.

On the theoretical side, many calculations of the H-dibaryon mass and structure have been performed using various models and theories. [1)–58)] Among them, the non-relativistic quark cluster model (QCM), which is successful in describing the baryon mass spectra and the experimental data of the nucleon–nucleon (NN) [179)] and nucleon–hyperon (NY) [22), 24)] scattering, is applied to the H-dibaryon state. [22)–31)] Theoretical analyses of the production [55), 130)–144)] of the H-dibaryon in various processes have been performed mainly by the coalescence model. The weak decay processes of the H-dibaryon are also studied.[*)] Donoghue *et al.* calculated the H-dibaryon lifetime. [69), 70)] For Cygnus X-3 events [72)] and ultra-high energy cosmic ray (UHECR) events [52)] beyond the GZK cutoff, [195), 196)] the H-dibaryon, with its mass below NΛ for Cygnus X-3 [72)] and below NN for UHECR, [52)] has been proposed as a possible long-lived, neutral particle which can reach the Earth without altering its direction by interstellar magnetic fields and losing its energy by the interaction with the cosmic background radiation. However, the calculated H-dibaryon lifetime is too short to explain the Cygnus X-3 events. [69), 70)] The H-dibaryon mass is now known to be higher than ΛN threshold, which makes the lifetime even shorter by many orders of magnitude and excludes the hypothesis for the UHECR events.

The H-dibaryon may exist in another environment e.g. in a double hypernucleus or in some special astrophysical objects. Double hypernucleus data have important meaning for the existence of the H-dibaryon in the sense that the binding energy of two Λ's is related to the lower limit of the H-dibaryon mass. However, whether $S = -2$ component in a double hypernucleus takes the form of $\Lambda\Lambda$ is not a trivial problem, and it is possible that a double

[*)] Weak decay process of a hypothetical $^1S_0$ bound state of two Λ's is also discussed by Krivoruchenko and Shchepkin. [71)] hypernucleus is an H-nucleus state.[166),158)] The possibility that H-dibaryon matter may exist in the core of a neutron star has also been pointed out.[162)]

In this article, we will review the studies of the H-dibaryon using the non-relativistic quark cluster model. In the next section, we will briefly summarize the theoretical and experimental status of the H-dibaryon. In §3, studies on the mass and structure of the H-dibaryon employing the quark cluster model will be reviewed. Though many of them are devoted to the baryon–baryon interaction including the $S = -2$, $J = T = 0$ channel, we will confine ourselves to the H-dibaryon state. In §4, a study on the interaction between a nucleon and an H-dibaryon will be reviewed. This NH interaction is used for an investigation on the property of the H-dibaryon in nuclear matter. The implication on the double hypernuclei will be also mentioned. In §5, we will review a study on the interaction between two H-dibaryons and the expected properties of H-dibaryon matter are discussed.

§2. H-dibaryon

In this section, the present status of theoretical and experimental studies on the H-dibaryon is briefly reviewed. Double hypernuclei, which have close connection with the H-dibaryon, are also reviewed.

2.1. *Theoretical status of the H-dibaryon*

Since Jaffe's prediction,[1)] many theoretical calculations have been made to predict the mass of the H-dibaryon, employing various QCD-inspired models (bag model[2)-12)], non-relativistic quark cluster model[22)-31)], Skyrme model[13)-21)], and so on[43)-54)]), QCD sum rule[37),38)] and lattice QCD[32)-36)]. Many of them predict the bound state. However, the results of calculations spread over wide range as shown in Fig. 1. It makes clear contrast to the success in reproducing the mass spectrum of mesons and baryons using the above methods.

This situation indicates an importance of the H-dibaryon as a touchstone of the predictive power of models and theories, and, on the other hand, experimental studies on the existence of the H-dibaryon are highly encouraged and will give a new insight into the dynamics of multiquark systems.

The basic mechanism which is expected to give large attractive force between quarks in the H-dibaryon is the color magnetic interaction (CMI), which is proportional to $\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j$, where $\boldsymbol{\sigma}_i$ is the Pauli matrix for the spin SU(2) group and $\boldsymbol{\lambda}_i$ is the Gell-Mann matrix for the color SU(3) group of the ith quark. In the flavor SU(3) symmetric limit, the expectation value of the sum of the operator $\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j$ for all pairs of quarks in a color singlet n-quark

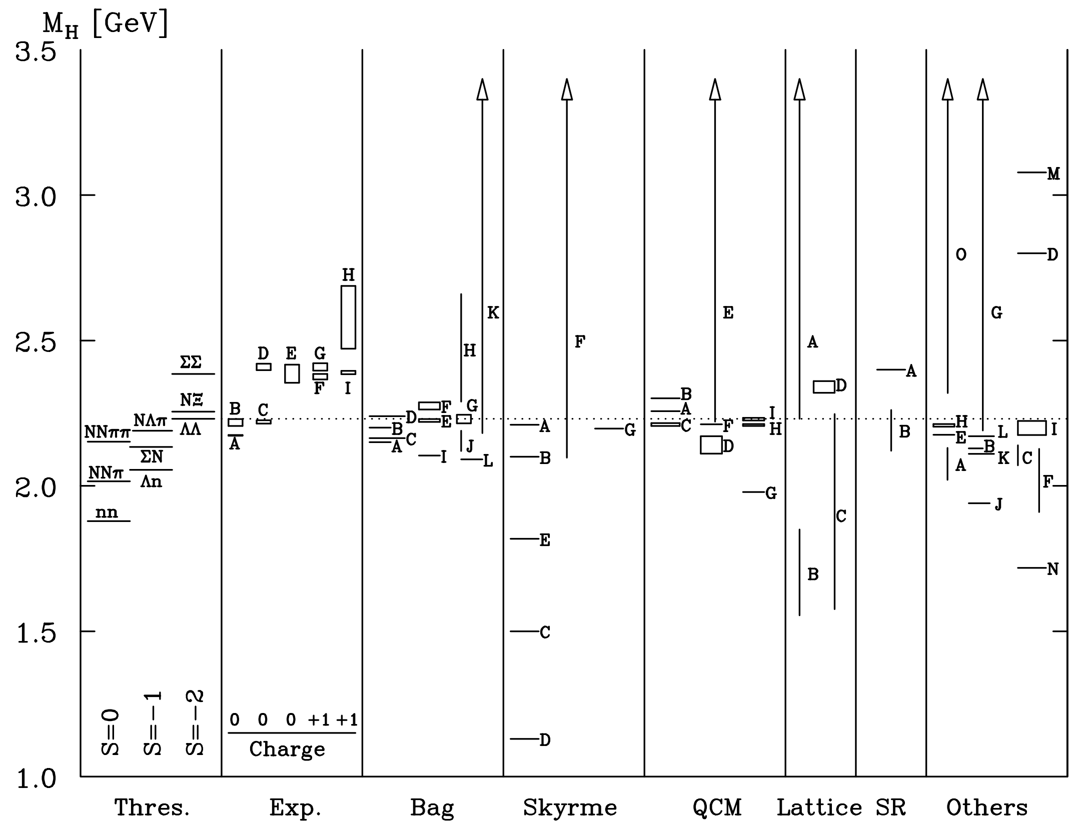

Fig. 1. The calculated masses of the H-dibaryon. The dotted line indicates the $\Lambda\Lambda$ threshold. Some thresholds [187)] (Thres.) and experimental masses (Exp.) of the H-dibaryon candidates reported so far are also shown. Vertical lines above the $\Lambda\Lambda$ threshold with upward arrow means that the upper limits are not shown in the literature. Corresponding references are as follows. Experiment (Exp.): A:79), B:80), C:84), D–G:81), H,I:83). The events D–I are claimed to be excited states. Bag model (Bag): A:1), B:2), C:3), D:4), E:5), F:6), G:7), H:8), I:9), J:10), K:11), L:12). Skyrme model (Skyrme): A:13), B:14), C:15), D:16), E:18), F:20), G:21). The value in Ref.19) is 3.9∼4.4 GeV. Non-relativistic quark cluster model (QCM): A:22), B:23), C:24), D:25), E:26), 27), F:28), G:29), H:30), I:31). Lattice gauge theory (Lattice): A:32), B:33), C:34), D:35). The value in Ref.36) is 2221±141 MeV for smaller lattice and heavier for larger lattice. QCD sum rule (SR): A:37), B:38). Others: A:39), B:40), C:41), D:42), E:43), F:44), G:9), H:45), I:46), J:47), K:48), L:49), M:50), N:52), O:53). Here we cite the values in the references as they are. Some of them are better to be compared with the mass of two Λ's calculated in the same framework. When the cited value is the binding energy, it is subtracted from the experimental value of the $\Lambda\Lambda$ threshold (2231 MeV). For further details please see each reference.

system with negative sign is

$$\Theta \equiv -\langle \sum_{i<j} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \rangle = n(n-10) + \frac{4}{3}J(J+1) + \left\langle \left(\sum_i \boldsymbol{f}_i \right)^2 \right\rangle , \qquad (2\cdot1)$$

where J is the spin of the n-quark system and

$$\left\langle \left(\sum \boldsymbol{f}_i\right)^2 \right\rangle = \frac{4}{3}\left(\lambda^2 + \lambda\mu + \mu^2\right) + 4(\lambda + \mu). \tag{2·2}$$

Here, (λ, μ) specifies the irreducible representation of the flavor SU(3) group. Θ favors the spin-flavor symmetric state. In fact, the expectation value of the Casimir operator $\left\langle (\sum \boldsymbol{f}_i)^2 \right\rangle$ is 12 for $[21]_{\mathrm{f}} = (1,1)$ state [181] and 0 for the flavor-singlet state like the H-dibaryon, so that $2\Theta = -16$ for two Λ's and $\Theta = -24$ for the H-dibaryon. This is the main reason why the H-dibaryon is likely to be a bound state. In terms of baryon configuration, the flavor SU(3) symmetric state $[222]_{\mathrm{f}}$ can be written as

$$|\,\mathrm{H}\rangle = \sqrt{\frac{1}{8}}\,|\,\Lambda\Lambda\rangle + \sqrt{\frac{4}{8}}\,|\,\mathrm{N}\Xi\rangle - \sqrt{\frac{3}{8}}\,|\,\Sigma\Sigma\rangle. \tag{2·3}$$

Recent QCD sum rule calculation proposed another possibility of the mass spectrum of the "H-dibaryons"[*], i.e. 27plet $[42]_{\mathrm{f}}$ and octet $[321]_{\mathrm{f}}$ H-dibaryons are lighter than the singlet H-dibaryon although they are resonances between $\Lambda\Lambda$ and $N\Xi$ thresholds in their calculation [68].

2.2. *Experimental status of the H-dibaryon*

Experimental searches of H-dibaryon have been performed elaborately. However, there have been no conclusive results on the existence of the H-dibaryon. The H-dibaryon searches have been made by several methods and in the wide range of its expected mass and lifetime. The H-dibaryon may be produced via (K^-, K^+) reaction, Ξ^--capture, heavy ion collision, $\bar{p}$-nucleus annihilation reaction and so on, and then the fragments produced in the decay of the H-dibaryon are trucked, e.g. for the H-dibaryon below $\Lambda\Lambda$ threshold, $\mathrm{H} \to \Sigma^- p$, $\mathrm{H} \to \Sigma^0 n$, $\mathrm{H} \to \Lambda n$, $\mathrm{H} \to p\pi^-\Lambda$, and so on. We do not step into the details of each experiment here. Instead, we list in Table I the experiments with their key reaction processes, i.e. how to produce the H-dibaryon, the decay processes to detect, or the process like $pp \to K^+K^+X$ in which the invariant mass analysis is made. Many of them are still in progress. See also the review paper by Paul [126] including ealier experiments [74),75),76] not shown in Table I and recent reviews by Ashery [128] and Klingenberg [129].

Most of the groups have reported that no evidence was found for the H-dibaryon. However, Shahbazian et al. [79),80),81),82),83] reported some H-dibaryon candidates including H-dibaryon excited states. Alekseev *et al.* [84] also reported two candidate events of the H-dibaryon with $M_{\mathrm{H}} = 2217.1 \pm 7.1$ MeV and 2224.3 ± 8.4 MeV. (See Exp. part in Fig. 1.

[*] The term H-dibaryon is used for the flavor-singlet H-dibaryon state in this paper. For other multiplets of uuddss color-singlet states, [1),2),47),59)-68),102] we refer the flavor multiplets they belong. The unitary transformation coefficients between each multiplets and two-baryon configurations are shown in Table 3 of Ref. 27).

Table I. Experimental searches for the H-dibaryon. For some of the experiments, we show in the third column to what range of the H-dibaryon mass that experiment is sensitive. For KEK E224, (pp) and (p) mean a proton pair and a proton in ^{12}C, respectively. $B_{\rm H} \equiv 2M_{\Lambda} - M_{\rm H}$.

Collaboration	reaction process (production/decay)	sensitive mass range
BNL E703 [77]	$p + p \to K^+ + K^+ + X$	$M_{\rm H} = 2.0 \sim 2.5$ GeV
BNL E810 [86,87,104]	Si+Pb collision / $\rm H \to \Sigma^- p, \Lambda p \pi^-$	
BNL E813 [88,89,90,91,92,103,104,106]	$K^- + p \to K^+ + \Xi^-$, $(\Xi^- {\rm d})_{\rm atom} \to {\rm H} + n$	$-15 < B_{\rm H} < 80$ MeV
BNL E830 [105]	$K^- + {}^3{\rm He} \to K^+ + {\rm H} + n$	
BNL E836 [90,91,92,93,103,104,106]	$K^- + {}^3{\rm He} \to K^+ + {\rm H} + n$ $K^- + {}^6{\rm Li} \to K^+ + {\rm H} + X$	$B_{\rm H} = 50 \sim 380$ MeV
BNL E864 [104,105]	Au+Pb collision	
BNL E885 [92,94,95,104]	$K^- + (p) \to K^+ + \Xi^-$, $(\Xi^- A)_{\rm atom} \to {\rm H} + X$ $K^- + A \to K^+ + X + {\rm H}$	
BNL E886 [96,104]	Au+Pt collision	
BNL E888 [97,98,99,104,106]	$p + A \to {\rm H} + X$ / $\rm H \to \Lambda n$ or $\Sigma^0 n$, ${\rm H}{+}A \to \Lambda + \Lambda + A$	$M_{\rm H} < 2150$ MeV
BNL E896 [100,104,105]	Au+Au collision / $\rm H \to \Sigma^- p \to n \pi^- p$, $\rm H \to \Lambda p \pi^- \to p \pi^- p \pi^-$, $\rm H \to \Lambda n \to p \pi^- n$	
BNL E910 [101]	$p{+}A$ / $\rm H \to \Lambda p \pi^-$, $\rm H \to \Sigma^- p$	
BNL STAR [125,102]	Au+Au collision	
KEK E176 [107,108,109,115]	$K^- + (pp) \to K^+ + {\rm H}$ $K^- + p \to K^+ + \Xi^-$, $\Xi^- + (p) \to {\rm H}$	
KEK E224 [110,111,112,113,114,115]	$K^- + (pp) \to K^+ + {\rm H}$ $K^- + (p) \to K^+ + \Xi^-$, $\Xi^- + (p) \to {\rm H}$	
KEK E248 [116]	$p + p \to K^+ + K^+ + X$	
Fermilab E791 [119]	${\rm H} \to p + \pi^- + \Lambda$, $\Lambda \to p + \pi^-$, ${\rm H} \to \Lambda + \Lambda \to p + \pi^- + p + \pi^-$	
Fermilab KTeV Collab. [120]	p+A / ${\rm H} \to p + \pi^- + \Lambda$	$M_{\rm H} = 2194$ ~ 2231 MeV
Shahbazian *et al.* [79,80,81,82,83]	$p + {}^{12}{\rm C} \to {\rm H}({\rm H}^+) + X$ / ${\rm H} \to \Sigma^- + p$, $\Sigma^- \to \pi^- n$ ${\rm H}^+ \to p + \pi^0 + \Lambda$, $\Lambda \to p + \pi^-$ ${\rm H}^+ \to p + \Lambda$, $\Lambda \to p + \pi^-$	
Alekseev *et al.* [84]	$n + A \to {\rm H} + X$ / ${\rm H} \to p\pi^-\Lambda$, $\Lambda \to p\pi^-$	
DIANA Collab. [117,118]	$\bar{p} + {\rm Xe} \to K^+{\rm H}X$, $K^+K^+{\rm H}X$ / ${\rm H} \to \Sigma^- + p$	
Condo *et al.* [78]	$\bar{p} + A \to {\rm H} + X$ / ${\rm H} \to \Sigma^- + p$	
Ejiri *et al.* [85]	${\rm d} \to {\rm H} + \beta + \nu$, ${}^{10}{\rm Be} \to {}^8{\rm Be} + {\rm H}$, ${}^{72}{\rm Ge} \to {}^{70}{\rm Ge} + {\rm H} + \gamma$, ${}^{127}{\rm I} \to {}^{125}{\rm I} + {\rm H} + \gamma$, ${}^{127}{\rm I} \to {}^{125}{\rm Te} + {\rm H} + \beta^+ + \nu$	$M_{\rm H} < 1875.1$ MeV
CERN NA49 [121]	Pb+Pb collision / ${\rm H} \to \Sigma^- p$, $\Lambda p \pi$	
CERN WA89 [122]	$\Sigma^- + A \to X + {\rm H}$ / ${\rm H} \to \Lambda\Lambda, {\rm N}\Xi$, ${\rm H} \to \Lambda p \pi^-, \Sigma^- p, \Sigma^0 n, \Lambda n$	
CERN WA97 [123]	Pb+Pb collision	
CERN ALICE [125]	Pb+Pb collision	
CERN OPAL [124]	Z^0 decay	

F, G, H and I are positively charged, and claimed to be a member of isotriplet together with neutral particle events, D and E.) These events, however, are controversial and more careful background event analysis and further experimental supports are needed. Anyway, few events with different masses cannot lead to any conclusion. Recently, the enhancement of the $\Lambda\Lambda$ invariant mass near threshold was observed in $^{12}\mathrm{C}(K^-, K^+\Lambda\Lambda)$ reaction (KEK E224).[114] However, according to the analysis by Ohnishi et al.,[146] this enhancement can be reproduced by the attractive $\Lambda\Lambda$ final state interaction with the possible scattering length corresponding to the bound or unbound $\Lambda\Lambda$. At present, whether this enhancement means a $\Lambda\Lambda$ resonance state or not is unclear.

Some of the H-dibaryon production processes are analyzed with theoretical models, and the H-dibaryon formation cross sections are calculated. In Table II, such analyses are listed. Some of them have been very helpful to experiments. For instance, the analyses of the

Table II. H-dibaryon production processes analyzed theoretically. *) Formation of a bound state of two Λ's as a doorway to the H-dibaryon state.

Author/Year	Ref.	process	
Badalyan *et al.* '82	55)	$pp \to K^+K^+\mathrm{H}$	
Aerts & Dover '82,'83	130), 131)	$K^- +^3\mathrm{He} \to K^+ + n + \mathrm{H}$	
Aerts & Dover '84	132)	$K^- + p \to K^+ + \Xi^-$,	$(\Xi^- p)_{\mathrm{atom}} \to \mathrm{H} + \gamma$
			$(\Xi^-\mathrm{d})_{\mathrm{atom}} \to \mathrm{H} + n$
			$(\Xi^{-4}\mathrm{He})_{\mathrm{atom}} \to \mathrm{H} + \mathrm{t}$
Dover *et al.* '89	133)	Si+Au collision	
Sano *et al.* '89	134)	Ne+Ne, p+Ne collision	
Kishimoto '89	135)	${}^{\mathrm{A}}_{\Lambda}\mathrm{N} \to^{\mathrm{A}-2}\mathrm{N} + \mathrm{H} + \pi^+$	
Dover *et al.* '91	136)	high-energy nuclear collision	
Moinester *et al.* '92	137)	$\Xi^- + p \to \pi^0 + \mathrm{H}, \rho^0 + \mathrm{H}$	
		$\Sigma^- + p \to K^0 + \mathrm{H}, K^{0*} + \mathrm{H}$	
		$\Lambda + p \to K^+ + \mathrm{H}, K^{*+} + \mathrm{H}$	
		$\Xi^- + p \to \mathrm{H} + \bar{p} + X$	
Aizawa & Hirata '92	138)	$K^- +^3\mathrm{He} \to K^+ + n + \mathrm{H}$	
Rotondo '93	139)	p-A collision	
Baltz *et al.* '94 *)	145)	Au+Au $\to (\Lambda\Lambda)_{\mathrm{b}} + X$	
Cole *et al.* '95	140)	p-A collision	
Cousins & Klein '97	142)	p-Pt collision	
Batty *et al.* '99	143)	$(\Xi^-\mathrm{d})_{\mathrm{atom}} \to \mathrm{H} + n$	
Kahana '99	144)	Au+Au collision	

H-dibaryon formation processes of BNL E813 and E836 are given in the papers by Aerts and Dover,[130),131),132] and have given guidances to the experiments through the direct comparison between theoretical prediction and experimental data. The relation between the lifetime and the mass of the H-dibaryon calculated by Donoghue *et al.*[69),70] also has played important roles in setting the searching mass and lifetime region in experiments. However, as May pointed out,[158] this relationship may depend critically on the wave function of the H-dibaryon.

2.3. *Double hypernuclei*

The double hypernucleus sheds light on the problem of the existence of the H-dibaryon from other aspects. Consider a double hypernucleus which is formed by the fusion of a non-strange nucleus and two Λ's with its binding energy $B_{\Lambda\Lambda} \equiv M(^{A-2}Z)+2M_\Lambda - M(^{A}_{\Lambda\Lambda}Z)$. If the mass of the double hypernucleus is heavier than the sum of the masses of the H-dibaryon and the original nucleus, then the double hypernucleus can decay strongly into the H-dibaryon and the original nucleus. Therefore, the existence of a double hypernucleus means that the mass of the H-dibaryon should be heavier than the mass of the two Λ's minus the binding energy: $M_\mathrm{H} > 2M_\Lambda - B_{\Lambda\Lambda}$. There have been reported a few events of double hypernuclei. Old nuclear emulsion experiments reported $^{10}_{\Lambda\Lambda}\mathrm{Be}$ [147] and $^{6}_{\Lambda\Lambda}\mathrm{He}$.[148] The former was reanalyzed by Dalitz *et al.*[159] An emulsion-counter hybrid experiment (KEK E176) reported that an event [149] can be interpreted as $^{10}_{\Lambda\Lambda}\mathrm{Be}$ [149), 160)] or $^{13}_{\Lambda\Lambda}\mathrm{B}$.[149), 160), 161)] Since then several candidates of double hypernuclei have been reported and the analyses are under way.[150), 151)] Many projects for double hypernucleus hunting are going on.[92), 95), 105), 106), 116), 127), 152) - 158)] In Table III, we list the double hypernuclei reported so far with their binding energy of two Λ's in the double hypernuclei $B_{\Lambda\Lambda}$ and the $\Lambda\Lambda$ interaction energy defined by $\Delta B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z) \equiv B_{\Lambda\Lambda}(^{A}_{\Lambda\Lambda}Z) - 2B_\Lambda(^{A-1}_{\Lambda}Z)$, where $B_\Lambda(^{A}_{\Lambda}Z) \equiv M(^{A-1}Z) + M_\Lambda - M(^{A}_{\Lambda}Z)$.

Table III. Reported double hypernuclear events are listed with their two Λ binding energy $B_{\Lambda\Lambda}$ and their $\Lambda\Lambda$ interaction energy $\Delta B_{\Lambda\Lambda}$. $\Delta B_{\Lambda\Lambda}$ for the event reported by Aoki *et al.* is cited from Refs.160), 161).

Year	Authors	Nuclide	$B_{\Lambda\Lambda}$ (MeV)	$\Delta B_{\Lambda\Lambda}$ (MeV)
1963	Danysz *et al.*[147]	$^{10}_{\Lambda\Lambda}\mathrm{Be}$	17.7 ± 0.4	4.3 ± 0.4
1966	Prowse *et al.*[148]	$^{6}_{\Lambda\Lambda}\mathrm{He}$	10.9 ± 0.5	4.7 ± 1.0
1991	Aoki *et al.*[149]	$^{10}_{\Lambda\Lambda}\mathrm{Be}$	8.5 ± 0.7	-4.9 ± 0.7
		or		
		$^{13}_{\Lambda\Lambda}\mathrm{B}$	27.6 ± 0.7	4.9 ± 0.7

After all, only the H-dibaryon with the small binding energy less than a few tens MeV survives in the present status.

§3. H-dibaryon in the quark cluster model

In this section, we will review the work on the H-dibaryon in the non-relativistic quark cluster models (QCMs). These investigations have been done in relation to $S = -2$, $J^\pi = 0^+$, $T = 0$ channel baryon–baryon interaction. The non-relativistic quark cluster models are successful in reproducing baryon mass spectrum and NN scattering phase shifts. Although the NY scattering data are not enough to make phase shift analyses, QCMs are also successful in reproducing the cross sections. We will not go further into the baryon spectrum and baryon–baryon interaction in QCM, and leave them to the other reviews in this supplement and a previous review [181] by Shimizu. We only mention here that the results for baryon–baryon interaction by the QCMs agree well not only qualitatively but also quantitatively. In

QCM, the internal wave function of a baryon is represented by a shell model wave function (usually the one of the harmonic oscillator), which is assumed to be known. Then, the total wave function of a baryon–baryon system is constructed by the antisymmetrized product of two baryons supplemented by the wave function of the relative motion between two baryons. The variational principle for energy (bound states) [182),184)] or S-matrix (scattering states) [185)] leads to the resonating group method (RGM) equation, by which the relative motion wave function is solved. The versions of QCMs are different in the way of reflecting the flavor symmetry breaking to the wave function, interaction between quarks, the treatment for meson exchange contributions, or the treatment of quark confinement.

The wave functions for the flavor SU(3) octet baryons with spin $\frac{1}{2}$ (N, Σ, Ξ, Λ) are given by

$$\phi(q^3) = \varphi(\boldsymbol{\xi}_1, \boldsymbol{\xi}_2)\mathcal{F}([21]_f)\mathcal{S}([21]_\sigma)\mathcal{C}([111]_c), \tag{3·1}$$

where $[21]_f$, $[21]_\sigma$ and $[111]_c$ denote the irreducible representations of the flavor, spin and color symmetry, respectively. For the low-lying baryons, the orbital part is assumed to have [3] symmetry. Most of the calculations assume the flavor SU(3) symmetry in the wave functions, although the symmetry breaking is taken into account in the hamiltonian. A few versions of QCMs take account of the mass difference between the non-strange and strange quarks also in the wave function. The orbital part of the single quark wave function in the baryon is taken to be Gaussian, i.e.

$$\varphi(\boldsymbol{r}) = (\pi b^2)^{-3/4} e^{-\frac{r^2}{2b^2}}, \tag{3·2}$$

where b is the size parameter of the harmonic oscillator. The flavor SU(3) symmetry breaking in $\varphi(\boldsymbol{r})$ is taken into account by the replacement: [24),9),45)]

$$\frac{1}{b_{\rm s}^2} = \frac{m_{\rm s}}{m_{\rm u,d}}\frac{1}{b^2} \tag{3·3}$$

for s quark. This replacement follows from the equilibrium of the kinetic energy of the quarks in hadrons, i.e.

$$\frac{3}{4m_{\rm u,d}b^2} = \frac{3}{4m_{\rm s}b_{\rm s}^2}. \tag{3·4}$$

The total wave function of baryon–baryon system is then written as

$$\Psi = \mathcal{A}[\varphi_1\varphi_2\chi], \tag{3·5}$$

where $\mathcal{A}$ is the antisymmetrization operator and χ is the wave function of the relative motion between two baryon clusters.

For the interaction between quarks, the one-gluon-exchange potential (OGEP) of the Fermi-Breit type[170] is used. Its form is

$$V_{ij}^{\mathrm{OGEP}}(\boldsymbol{r}_{ij}) = \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \frac{\alpha_{\mathrm{s}}}{4} \left[\frac{1}{r_{ij}} - \frac{\pi}{m_i m_j} \left(1 + \frac{2}{3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) \delta(\boldsymbol{r}_{ij}) \right]$$
$$- \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \frac{\alpha_{\mathrm{s}}}{4} \frac{1}{2 m_i m_j} \left(\frac{\boldsymbol{p}_i \cdot \boldsymbol{p}_j}{r_{ij}} + \frac{\boldsymbol{r}_{ij} \cdot (\boldsymbol{r}_{ij} \cdot \boldsymbol{p}_i) \boldsymbol{p}_j}{r_{ij}^3} \right), \qquad (3{\cdot}6)$$

where $\boldsymbol{r}_{ij} = \boldsymbol{r}_i - \boldsymbol{r}_j$ is the relative coordinate between quarks, and $\boldsymbol{p}_i$ and $\boldsymbol{p}_j$ are the momenta of the quarks. In the right-hand side of (3·6), the first term is momentum-independent while the second term is the momentum-dependent term. The part proportional to $\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$ in the first term is the color-magnetic interaction (CMI) term, which gives a large attraction for the H-dibaryon state.

The flavor SU(3) symmetry breaking (FSB) is implemented in several ways. It appears in the following parts: (i) the wave function; (ii) the quark mass term; (iii) the kinetic term; (iv) OGEP. The FSB in the quark mass term causes only constant shifts. In order to avoid complications, (i) and (iii) are often treated as flavor symmetric, and all the effects from FSB is ascribed to (iv). The essential results are the same for the baryon spectra and the NY scattering phase shifts, independently of the way FSB is taken into account.

Takeuchi and Oka[26,27] studied another type of interquark force, which is induced by light-quark–instanton coupling and therefore named the instanton induced interaction (III). [188,189] III was originally derived by 't Hooft to represent the effect of the $\mathrm{U_A}(1)$ symmetry breaking.[188] Takeuchi and Oka reduce III to a nonrelativistic form, which consists of the two- and three-body forces.[190,26,27,73] The two-body interaction terms include a color-magnetic term, which has the same spin-color structure as that in OGEP. The three-body interaction has interesting features. It acts among u-d-s quarks, but in a single baryon it is inactive so that only the two-body part of III contributes to the mass splittings among baryons. The three-body part of III acts as a strong short-range repulsive force between strange baryons. If we assume that a part of CMI stems from the two-body part of III, the hadron mass spectra are hardly affected except for the η–η' splitting. In the H-dibaryon, however, the short-range repulsion due to the three-body part of III may reduce or even cancel the attractive force due to CMI. Unfortunately, ambiguity in determining the strength of the three-body force of III prevents us from drawing a definite conclusion. However, it is interesting to point out that, for the spectrum of dibaryons with strangeness, there may be a driving force other than OGEP.

One comment is in order. Morimatsu and Takizawa[57] estimated the effects of a $\mathrm{U_A}(1)$

breaking interaction of the form:[191)]

$$\mathcal{L}_6 = -6K\left\{\det(\bar{\psi}_{\rm R}\psi_{\rm L}) + ({\rm h.c.})\right\}, \tag{3·7}$$

where ψ is the quark field and K is a coupling constant determined so as to reproduce the mass spectra of the pseudoscalar meson nonet reasonably. Using the wave functions of the MIT bag model or the nonrelativistic quark model, they calculated matrix elements of the $\mathrm{U_A}(1)$ breaking interaction for various baryon and two-baryon channels. They found that for the H-dibaryon the three-body part of the interaction (3·7) also gives a repulsion, but that its magnitude is much less than the result of Takeuchi and Oka.[26), 27)] The two-body part gives strong attraction ($-36 \sim -86$ MeV), while the three-body part gives somewhat moderate repulsion ($4 \sim 17$ MeV).[*)]

Though the OGEP is known to reproduce the short-range part of the NN phase shift successfully, the medium- and long-range part needs the meson-exchange contribution. The simplest phenomenological treatment is to include the effective meson exchange potential (EMEP) as an interaction potential between baryons.[179)] More elaborate studies to include the meson exchange as an interaction between quarks have been done although mesons are also composites of quark-antiquark.

The earlier study by Oka *et al.*[22)] focused mainly on the short-range part of the baryon–baryon interaction, and the meson-exchange was beyond the scope of it. The first study that applied the meson-exchange potential to the H-dibaryon channel was by Straub *et al.*[24)] They include the pseudoscalar-meson exchange potential on the quark level,

$$\begin{aligned} V_{ij}^{\rm psM} &= -\frac{1}{3}\frac{g_{\rm qqM}^2}{4\pi}\frac{\mu^2}{4M_AM_B}\frac{\Lambda^2}{\Lambda^2-\mu^2}\exp(-\mu^2b^2/3) \\ &\quad \times\mu\left(\frac{\exp(-\mu r_{ij})}{\mu r_{ij}} - \frac{\Lambda^3}{\mu^3}\frac{\exp(-\Lambda r_{ij})}{\Lambda r_{ij}}\right)\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j O_{ij}^{\rm F}, \end{aligned} \tag{3·8}$$

where $g_{\rm qqM}$ is the coupling constant, μ is the meson mass, M_A and M_B are the masses of two baryon clusters, Λ is the cutoff parameter and $O_{ij}^{\rm F}$ is the flavor operator, and the phenomenological σ-meson exchange potential on the baryon level,

$$\begin{aligned} V_\sigma(r) = -\frac{g_\sigma^2}{4\pi}\frac{1}{2m_\sigma^2R_\sigma^2 r}&\{[1-\exp(-m_\sigma r)-\exp(-2m_\sigma R_\sigma)\sinh(m_\sigma r)]\theta(2R_\sigma - r) \\ &+[\cosh(2m_\sigma R_\sigma)-1]\exp(-m_\sigma r)\theta(r-2R_\sigma)\}, \end{aligned} \tag{3·9}$$

where R_σ is the cutoff radius of the σ-meson.[180)] They found that the σ-meson exchange gives quite important attractive contribution. Only with the one-gluon-exchange potential

*) The instanton contribution on H-dibaryon mass is estimated also in the earlier work of Ref. 12). The contribution from three-body force is very small (about 3 MeV) in that calculation.

and the pseudoscalar meson exchange, the H-dibaryon does not appear as a bound state, but a resonance is found at $E_{\rm cm} = 26$ MeV in the $\Lambda\Lambda$-NΞ-$\Sigma\Sigma$ three channel phase shift calculation. When the σ-meson exchange potential is switched on, the H-dibaryon mass is pushed down below the $\Lambda\Lambda$ threshold with its binding energy $15 \sim 25$ MeV according to the σ-meson coupling strength $g_\sigma^2/4\pi$, whose range is determined so as to reproduce the data of the ΛN and ΣN cross sections.

Nakamoto *et al.* use a more elaborate meson exchange potential, which incorporates the scalar-meson nonet instead of the flavor-singlet σ-meson.[28] In their $\Lambda\Lambda$-NΞ-$\Sigma\Sigma$ channel coupling calculation using one of their models RGM-F, an H-dibaryon bound state is found at 19 MeV below the $\Lambda\Lambda$ threshold although this value should be interpreted as a qualitative result because in their calculation the mass differences among $\Lambda\Lambda$, NΞ and $\Sigma\Sigma$ thresholds are smaller than the experimental values. Note that the contribution from the flavor-octet scalar-meson exchanges is repulsive for the H-dibaryon,[167] although the net contribution from the scalar-meson exchanges is attractive.

Chiral quark models, in which the interaction between constituent quarks is mediated by Goldstone bosons, or their hybrid versions describe successfully baryon spectra, NN and NY scattering, and properties of the deuteron.[51), 171) - 178] Stancu *et al.* argued that the H-dibaryon will not bind[50] because the pseudoscalar meson exchange gives rise to a strong repulsion in contrast to the one-gluon exchange. They estimated an adiabatic potential, which is defined as $V(R) = \langle H\rangle_R - \langle H\rangle_\infty$, where $\langle H\rangle_R$ is the expectation value of the hamiltonian with respect to the state with the separation distance R between two $(0{\rm s})^3$ clusters, and $\langle H\rangle_\infty = 2m_\Lambda$. They obtained $V(0)$=847 MeV and claimed that the H-dibaryon should not exist because of this strong repulsion.

Shimizu and Koyama studied more quantitatively the above chiral quark model and its hybrid version by an extended RGM, which allows baryons to swell in the interaction region.[30] The calculation is made for three cases of the interactions between quarks, which reproduce the NΔ mass difference correctly. The model (I) uses the OGEP with only long range Yukawa parts of the pseudoscalar meson exchange retained. The model (II) is a hybrid model, in which the pseudoscalar meson exchange gives about a third of the NΔ mass difference and the rest of the NΔ mass splitting is the contribution from the OGEP. For the meson exchange parts, the pseudoscalar meson exchange (PSME) potential $V_{ij}^{\rm ps}(r_{ij})$ and the σ meson exchange potential $V_{ij}^{\sigma}(r_{ij})$ are taken into account. They have the following forms:

$$V_{ij}^{\rm ps}(r_{ij}) = \frac{1}{3}\frac{g_c^2}{4\pi}\frac{m_{\rm ps}^2}{4m_i m_j}\frac{\Lambda^2}{\Lambda^2 - m_{\rm ps}^2}\boldsymbol{f}_i\cdot\boldsymbol{f}_j\sigma_i\cdot\sigma_j\left\{\frac{e^{-m_{\rm ps}r_{ij}}}{r_{ij}} - \left(\frac{\Lambda}{m_{\rm ps}}\right)^2\frac{e^{-\Lambda r_{ij}}}{r_{ij}}\right\}, \tag{3·10}$$

$$V^{\sigma}_{ij}(r_{ij}) = \frac{g_c^2}{4\pi}\frac{\Lambda^2}{\Lambda^2 - m_\sigma^2}\left(\frac{e^{-m_\sigma r_{ij}}}{r_{ij}} - \frac{e^{-\Lambda r_{ij}}}{r_{ij}}\right), \tag{3·11}$$

where $\boldsymbol{f}$ is the generators of the flavor SU(3) group. In model (II), the coupling constant g_c is determined so as to give the πNN coupling constant:

$$\frac{g_c^2}{4\pi}\frac{m_\pi^2}{4m_{\mathrm{u,d}}^2} = \left(\frac{3}{5}\right)^2 \frac{g_{\pi\mathrm{NN}}^2}{4\pi}\frac{m_\pi^2}{4M_{\mathrm{N}}^2}. \tag{3·12}$$

The model (III) has no OGEP part and the NΔ mass difference is wholly given by the PSME. The three coupling constants, $g_c(\pi)$, $g_c(K)$ and $g_c(\eta)$, for pseudoscalar meson exchanges are determined so as to give the NΔ mass difference and the thresholds NΞ and $\Sigma\Sigma$ correctly, whereas a common coupling constant is used in models (I) and (II). The results of the $\Lambda\Lambda$-NΞ-$\Sigma\Sigma$ coupled channel calculation are as follows. When the same σ-meson exchange potential is used for three models, the binding energies for model (I) and (II) are 74.5 MeV and 24.4 MeV, respectively, but there is no bound state for model (III). However, the model (II) and (III) are found to give less attractive NN potential. If the strengths of the σ-meson exchange potential are increased so as to reproduce the phase shifts of the NN $^1\mathrm{S}_0$ state, the binding energy for the model (II) becomes 56.8 MeV and even the model (III) gives a bound state with its binding energy 15.6 MeV. After all, the binding energies of the H-dibaryon in chiral (or hybrid chiral) quark models are much less than those in OGE models due to the repulsion mainly from the pion exchange, but room for the bound H-dibaryon still remains on account of the medium range attraction from the σ-meson exchange. The calculation also shows that the wave function of the H-dibaryon is more spread out than the simple $(0\mathrm{s})^6$ configuration due to the medium range attraction. The $\Sigma\Sigma$ component is drawn inside because this channel gains a lot of attractions at short distances due to the CMI, whereas the $\Lambda\Lambda$ and NΞ components are slightly apart from each other to reduce the relative kinetic energy. A recent study in a hybrid chiral quark model also gives similar results.[31)]

Quark confinement is realized in the QCMs by a potential among quarks. Mostly used one is the linear or quadratic type of potential between two quarks:

$$-a_1|\boldsymbol{r}_1 - \boldsymbol{r}_2|\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_2 \quad \text{(linear)} \tag{3·13a}$$

or

$$-a_2(\boldsymbol{r}_1 - \boldsymbol{r}_2)^2\boldsymbol{\lambda}_1\cdot\boldsymbol{\lambda}_2 \quad \text{(quadratic).} \tag{3·13b}$$

These types of confinement potential hardly affect the physical observables of the baryon–baryon system such as the binding energy (of two baryon system) or the phase shift. This is because in the flavor symmetric limit $\langle\sum_{i<j}\boldsymbol{\lambda}_i\cdot\boldsymbol{\lambda}_j\rangle$ is dependent only on the number of

quarks so that the confinement potential term does not give any net force between baryon clusters. Even when the flavor symmetry breaking is introduced in the wave function, only negligible contribution arises.

Another quark confinement mechanism called flip-flop model is proposed [192] and studied in $S = -2$ channel. [25] The flip-flop model avoids the long-range "color van der Waals force" [193] which appears through a virtual excitation of the hidden color state (the color octet dipole state). Such a force cannot be represented by a two-body potential and is necessarily a many-body force. Let us illustrate a two-baryon system to explain the flip-flop model. First, six quarks are divided into nearest-neighbor three quark systems. Then if each cluster is in the color-singlet state, both of three quark systems are confined separately, i.e. the confining force is inactive between the quarks in the different clusters. Otherwise the confining force acts among all pairs of six quarks.

Including the pseudoscalar and σ-meson exchange, Koike *et al.* found the H-dibaryon binding energy 64.4∼124.5 MeV varying with the coupling constant for the σ-meson. [25] This binding energy is rather large compared with other QCM calculations with the confinement through the two-body potential. There is a bound state for another channel with $S = -2$, i.e. $J = 1$, $T = 0$ NΞ channel with its binding energy 6.3 MeV, which can decay only via weak interaction.

To summarize the QCM for the H-dibaryon, we can extract some common results as follows. Here we consider the calculations using (OGEP)+(meson-exchange potential)+(two-body confinement potential) and including the FSB as a "standard calculation" in QCM.

(i) The CMI in the OGEP gives large attractive force but it is not sufficient for the H-dibaryon to bind. The scalar meson (σ-meson) exchange is indispensable. The coupled channel $\Lambda\Lambda$-NΞ-$\Sigma\Sigma$ calculation without σ-meson exchange potential shows a resonance in the $\Lambda\Lambda$ phase shift, [22), 24)] which disappears in the calculation with σ-meson exchange. [24]

(ii) The FSB reduces the attractive force due to the CMI. [22), 23), 28)] In the flavor SU(3) symmetric limit, the H-dibaryon bound states appear even without the σ-meson exchange. These bound states vanish when the effect of the FSB is taken into account in the calculation without σ-meson exchange. [22), 23)]

(iii) The $\Lambda\Lambda$-NΞ-$\Sigma\Sigma$ coupled, flavor-singlet structure is essential for the H-dibaryon to bind. The $\Lambda\Lambda$ single-channel calculation shows that the interaction between two Λ's has repulsive core. [22), 24), 28)] On the other hand, the NΞ and $\Sigma\Sigma$ single-channel calculations show the attractive interactions in these channels. This attractive nature leads to a resonance structure in the calculation without σ-meson exchange [22), 24)] or a bound state [24] in the full three-channel calculation.

(iv) Although the relative magnitudes of the $\Lambda\Lambda$, NΞ and $\Sigma\Sigma$ components of the bound

H-dibaryon wave function roughly agree with those of the flavor-singlet state of eq.(2·3), [22), 24), 25)] the wave function changes as a reflection of the natures of each components. [30)]

(v) The quark-quark interactions other than OGEP, i.e. III and Goldstone boson exchange, may reduce the attractive force in the H-dibaryon channel substantially. [26), 27), 50), 30), 31)]

§4. Nucleon–H-dibaryon interaction

In this section, we will review a study on the interaction between a nucleon and an H-dibaryon in Ref. 165) and further comment on the possible implication of the H-dibaryon to the world of the nucleus with $S = -2$. As stated in the introduction, the H-dibaryon is not only an interesting object in itself but also important in $S = -2$ sector nuclear physics. In fact, though a few events of double hypernuclei were reported [147), 148), 149)] and several candidate events have successively been reported recently, [150), 151)] structures of these double hypernuclei have not been fully understood yet. It is possible that there is a double hypernucleus which have the character of an H-nucleus rather than $\Lambda\Lambda$ nucleus, if the H-dibaryon is strongly bound in the nucleus.

In §4.1, the framework of the QCM commonly used in the calculation of the NH and HH (§5) interaction is explained. The results for the NH interaction [165)] are recapitulated in §4.2. In §4.3, a study on the behavior of the H-dibaryon in nuclear matter [166)] is reviewed. The relation with the $\Lambda\Lambda$ in nuclear matter is also discussed.

4.1. *The framework of the quark cluster model*

In this work and the work on the interaction between two H-dibaryons in the next section, the following framework of the quark cluster model is used.

(i) The orbital parts of the internal wave functions of the nucleon and the H-dibaryon have $(0s)^3$ and $(0s)^6$ configurations, respectively, and the flavor symmetry breaking (FSB) is not introduced in the wave function, i.e. FSB is reflected only in the hamiltonian. The internal wave functions of a nucleon $\phi_{\rm N}(\boldsymbol{\xi}_{\rm N})$ and an H-dibaryon $\phi_{\rm H}(\boldsymbol{\xi}_{\rm H})$ are then written as

$$\begin{aligned}\phi_{\rm N}(\boldsymbol{\xi}_{\rm N}) &= \varphi_{\rm N}(\boldsymbol{\xi}_{\rm N})\mathcal{C}([111]_{\rm C})\mathcal{S}([3]_{\rm SF}),\\ \phi_{\rm H}(\boldsymbol{\xi}_{\rm H}) &= \varphi_{\rm H}(\boldsymbol{\xi}_{\rm H})\mathcal{C}([222]_{\rm C})\mathcal{S}([222]_{\rm SF}),\end{aligned} \tag{4·1}$$

where $\boldsymbol{\xi}_{\rm N}$ and $\boldsymbol{\xi}_{\rm H}$ are internal coordinates. Here, $\varphi_{\rm N}$ and $\varphi_{\rm H}$ are the orbital parts, $\mathcal{C}$ is the color part and $\mathcal{S}$ is the spin-flavor part of the internal wave functions.

For the relative motion, only S-waves are considered.

(ii) The hamiltonian is

$$H = K + V, \tag{4·2}$$

where K is the sum of the kinetic energy of each quark with the center-of-mass kinetic energy of the totalsystem subtracted,

$$K = \sum_{i=1}^{N_q} \frac{\boldsymbol{p}_i^2}{2m_i} - \frac{(\sum \boldsymbol{p}_i)^2}{2\sum m_i}, \tag{4·3}$$

where N_q is the total number of quarks, and V is the potential term which consists of the residual interaction and the confinement term $V^{\rm conf}$. For the residual interaction, the momentum-independent part of the one-gluon-exchange potential of the Fermi-Breit type $V_{ij}^{\rm OGEP}$ (3·6) supplemented by meson exchange potential is used. The relevant part of $V_{ij}^{\rm OGEP}$ is

$$V_{ij}^{\rm OGEP} = \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j \frac{\alpha_{\rm s}}{4} \left[\frac{1}{r_{ij}} - \frac{\pi}{m_i m_j} \left(1 + \frac{2}{3} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) \delta(\boldsymbol{r}_{ij}) \right]. \tag{4·4}$$

Here, $\alpha_{\rm s}$ is the quark-gluon coupling constant. The color magnetic interaction term plays important role in producing the H-dibaryon bound state and short-range repulsive forces between NN, NH or HH. So FSB is introduced by the following parametrization:

$$\frac{\pi}{m_i m_j} \rightarrow \xi_{ij} \frac{\pi}{{m_u}^2} \tag{4·5}$$

so that the thresholds, $\mathit{\Lambda\Lambda}$, $N\mathit{\Xi}$ and $\mathit{\Sigma\Sigma}$ are correctly given.[22] Here, the parameter $\xi_{ij} = 1$ when both i and j are u- or d-quarks, $\xi_{ij} = \xi_1$ when either i or j is an s-quark, and $\xi_{ij} = \xi_2$ when both i and j are s-quarks. In the naive interpretation of (4·5), $\xi_2 = \xi_1^2$. However, as a consequence that all FSB effects are burdened to this term, the actual values are taken to be $\xi_1 = 0.6$ and $\xi_2 = 0.1$.[22] For the two-body confinement potential, quadratic type (3·13b) is used, i.e.

$$V^{\rm conf} = -a_2 \sum_{i<j} r_{ij}^2 \boldsymbol{\lambda}_i \cdot \boldsymbol{\lambda}_j. \tag{4·6}$$

(iii) The meson exchange contributions to NH and HH interactions are introduced as follows. An effective meson exchange potential (EMEP) is introduced into the RGM hamiltonian kernel,[179]

$$V_{\rm RGM}^{\rm EMEP}(\boldsymbol{R}, \boldsymbol{R}') = \int N_{\rm RGM}^{1/2}(\boldsymbol{R}', \boldsymbol{R}'') \mathcal{V}(R'') N_{\rm RGM}^{1/2}(\boldsymbol{R}'', \boldsymbol{R}) {\rm d}\boldsymbol{R}'', \tag{4·7}$$

where the phenomenological potential $\mathcal{V}(R)$ is taken to be gaussian[166),167]

$$\mathcal{V}(R) = V_0 \exp(-R^2/\alpha^2). \tag{4·8}$$

This is a simple extension of the method developed in the pioneering work of the NN interaction in the quark cluster model.[179] First, the range of the strength of EMEP for the

H-dibaryon, $V_0^{\rm H}$, is determined to give the mass of the H-dibaryon between $\Lambda\Lambda$ threshold and the lower limit determined from the KEK experiment,[149] i.e.

$$0 < 2M_\Lambda - M_{\rm H} < 27.6 \text{ MeV}. \tag{4·9}$$

Then, the gaussian size parameters, $\alpha_{\rm NH}$ and $\alpha_{\rm HH}$, and the strength of EMEP, $V_0^{\rm NH}$ and $V_0^{\rm HH}$, for the NH and HH interactions, respectively, are obtained by the direct convolution[165), 167)] of the EMEP for the H-dibaryon as

$$\alpha_{\rm NH}^2 = \alpha_{\rm H}^2 + \frac{1}{6}b^2, \qquad V_0^{\rm NH} = 2V_0^{\rm H}\left(\frac{\alpha_{\rm H}}{\alpha_{\rm NH}}\right)^3, \tag{4·10a}$$

$$\alpha_{\rm HH}^2 = \alpha_{\rm H}^2 + \frac{1}{3}b^2, \qquad V_0^{\rm HH} = 4V_0^{\rm H}\left(\frac{\alpha_{\rm H}}{\alpha_{\rm HH}}\right)^3. \tag{4·10b}$$

The parameters used in the calculations are listed in Table IV.

Table IV. The parameters used in the calculation of the NH and HH interactions.

m (MeV)	b (fm)	α_s	a_2 (MeV/fm^2)	ξ_1	ξ_2	$\alpha_{\rm NH}$ (fm)	$\alpha_{\rm HH}$ (fm)
300	0.6	1.39	33.0	0.6	0.1	0.97	1.00

$2M_\Lambda - M_{\rm H}$	$V_0^{\rm NH}$ (MeV)	$V_0^{\rm HH}$ (MeV)
0	−601	−1096
8.5	−623	−1136
27.6	−673	−1227
43.4	−714	−1302

4.2. *The property of the NH interaction*

In Fig. 2 the binding energy of the NH system as a function of the strength parameter of EMEP, $V_0^{\rm NH}$, is shown. The NH scattering phase shift δ is shown in Fig. 3 as a function of the relative energy $E_{\rm rel}$ for several $V_0^{\rm NH}$. The essential results of this calculation are as follows.

(i) The bound state exists only for the rather strong EMEP. For example, NH system is loosely bound with its binding energy 0.2 MeV for $V_0^{\rm NH} = -714$ MeV.

(ii) The fundamental character of the NH interaction is that it has a repulsive core in the short-range region and an attractive force in the medium-range region. This nature of the interaction is clearly seen in the behavior of the phase shift of Fig.3. The origin of the short-range repulsion is the Pauli principle for quarks and CMI. The normalization kernel, which represents the overlap of the wave function in a sense, is $\frac{5}{8}$ at zero separation of NH clusters, while one at infinity. Therefore the Pauli blocking is $\frac{3}{8}$ times as effective as the complete blocking. The role of CMI can be qualitatively explained as follows. In the flavor

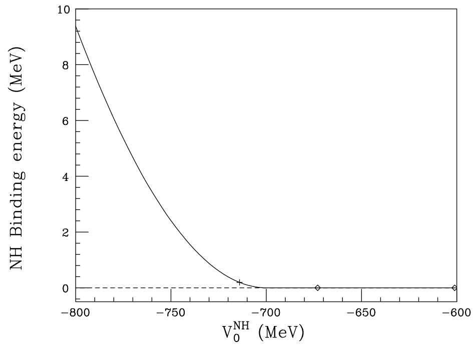

Fig. 2. The binding energy of the NH system as a function of the strength of EMEP V_0^{NH}. (Taken from Ref. 165).)

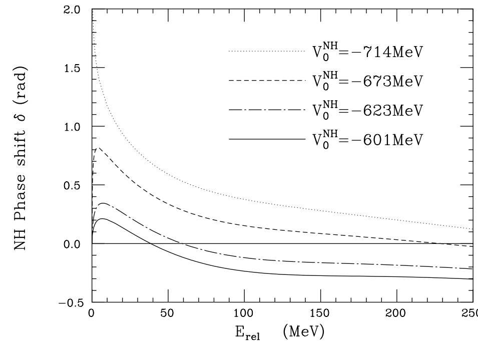

Fig. 3. The NH scattering phase shift δ as a function of the relative energy E_{rel} for several V_0^{NH}. (Taken from Ref. 165).)

SU(3) symmetric limit, the CMI term is proportional to Θ defined in (2·1). For a nucleon and an H-dibaryon, $\Theta_{\mathrm{N}} = -8$ and $\Theta_{\mathrm{H}} = -24$, respectively, while for an NH system of zero separation $\Theta_{\mathrm{NH}} = 4$. The difference $\Theta_{\mathrm{NH}} - (\Theta_{\mathrm{N}} + \Theta_{\mathrm{H}}) = 36$ causes a strong repulsive force. The main contribution to the medium-range attractive force is from the σ-meson exchange.

4.3. *H-dibaryon in nuclear matter*

In Ref. 166), the property of the H-dibaryon in nuclear matter is investigated. First, the single-particle potential of the H-dibaryon in nuclear matter is calculated employing the Brueckner theory. The non-local potential between N and H derived from the QCM (§4.2) is Fourier-transformed and then used to solve the Bethe–Goldstone equation. In this work, N and H are treated as elementary particles. Single-particle potential in uniform matter is well described by an effective mass and a depth of the potential. The effective mass of the H-dibaryon, M_{H}^*, and the potential well depth, D_{H}, are dependent on the EMEP parameter V_0^{NH}. The obtained values are listed in Table V.

Table V. The ratio of the H-dibaryon effective mass in nuclear matter, M_{H}^*, to the H-dibaryon mass in free space, M_{H}, and the well depth, D_{H}, in nuclear matter. V_0^{NH} is the strength of the effective meson exchange potential between a nucleon and an H-dibaryon. Those under the effect of the coupling with $\Lambda\Lambda$, $M_{\mathrm{H}}^{*(\mathrm{c})}$ and $D_{\mathrm{H}}^{(\mathrm{c})}$, are also shown for the case $\Delta M_{\mathrm{H}} = 0$.

V_0^{NH} (MeV)	$M_{\mathrm{H}}^*/M_{\mathrm{H}}$	D_{H} (MeV)	$M_{\mathrm{H}}^{*(\mathrm{c})}/M_{\mathrm{H}}$	$D_{\mathrm{H}}^{(\mathrm{c})}$ (MeV)
−601	0.853	14.5	0.848	7.2
−623	0.820	28.7	0.814	22.3
−673	0.769	63.6	0.769	60.0
−714	0.755	95.3	0.750	90.1

In the actual situation, the coupling with the $\Lambda\Lambda$ channel is important for the H-dibaryon near the $\Lambda\Lambda$ threshold. This effect is taken into account by a model with a simple coupling vertex function between H-dibaryon and $\Lambda\Lambda$,

$$\Gamma(k) = g e^{-b^2 k^2}. \tag{4·11}$$

The physical H-dibaryon mass $M_{\rm H}$ ($\Delta M_{\rm H} = M_{\rm H} - 2M_\Lambda$) is determined by the position of the pole of the propagator:

$$\Delta M_{\rm H} - \Delta M_{\rm H}^{(0)} - {\rm Re}[g^2 \Sigma(\Delta M_{\rm H})] = 0, \tag{4·12}$$

where $\Delta M_{\rm H}^{(0)}$ is the bare H-dibaryon mass measured from the $\Lambda\Lambda$ threshold and Σ is the self-energy defined by

$$g^2 \Sigma(E) = \int \frac{{\rm d}\boldsymbol{q}}{(2\pi)^3} \frac{\Gamma^2(q)}{E - q^2/M_\Lambda + i\epsilon}. \tag{4·13}$$

Eq.(4·12) determines $\Delta M_{\rm H}$ provided $\Delta M_{\rm H}^{(0)}$ and $\Gamma(q)$ are given. Here, inversely, eq.(4·12) is used to determine the bare mass $M_{\rm H}^{(0)}$ from a given $\Delta M_{\rm H}$. $\Gamma(k)$ is determined so as to reproduce the low-energy behavior of $\Lambda\Lambda$ scattering given by a QCM in Ref. 27). Then using $M_{\rm H}^{(0)}$ and $\Gamma(k)$, the properties of the H-dibaryon in nuclear matter affected by the coupling with the $\Lambda\Lambda$ channel are studied through the analysis of the H-dibaryon propagator in nuclear matter, $G_{\rm H}(E, P)$:

$$G_{\rm H}^{-1}(E, P) = E - \frac{P^2}{2M_{\rm H}^*} - (\Delta M_{\rm H}^{(0)} - D_{\rm H}) - \int \frac{{\rm d}\boldsymbol{q}}{(2\pi)^3} \frac{\Gamma(q)^2}{E - \frac{P^2}{4M_\Lambda^*} - \frac{q^2}{M_\Lambda^*} + 2D_\Lambda + i\epsilon}, \tag{4·14}$$

where P is the momentum of the H-dibaryon. The potential well depth of Λ in nuclear matter, D_Λ, used here is 27.5 MeV and $M_\Lambda^*/M_\Lambda = 0.8.$[186] Here, $M_{\rm H}^*/M_{\rm H}$ and $D_{\rm H}$ are regarded as the quantities for the "bare" H-dibaryon, which are not subject to the coupling. The total energy (= kinetic energy + potential energy) of the H-dibaryon in nuclear matter, $E_{\rm H}(P)$, obtained from the condition

$$ {\rm Re}[G_{\rm H}^{-1}(E_{\rm H}, P)] = 0, \tag{4·15}$$

is parametrized in the effective mass approximation:

$$E_{\rm H}(P) = \frac{P^2}{2M_{\rm H}^{*({\rm c})}} - D_{\rm H}^{({\rm c})}. \tag{4·16}$$

$M_{\rm H}^{*({\rm c})}/M_{\rm H}$ and $D_{\rm H}^{({\rm c})}$ for $\Delta M_{\rm H} = 0$ are also shown in Table V. The value of $D_{\rm H}^{({\rm c})}$ is to be compared with $2D_\Lambda$. If the energy of the H-dibaryon, $\frac{P^2}{2M_{\rm H}^{*({\rm c})}} - D_{\rm H}^{({\rm c})}$, is less than the $\Lambda\Lambda$ threshold energy, $2\left[\frac{(P/2)^2}{2M_\Lambda^*} - D_\Lambda\right]$, in nuclear matter, the H-dibaryon becomes the ground state of an $S = -2$ two-baryon system in nuclear matter. From the value of $D_{\rm H}^{\rm (c)}$ listed in Table V, the H-dibaryon does not appear as the bound state in nuclear matter and the ground state in nuclear matter is $\Lambda\Lambda$ for most of the plausible values of $V_0^{\rm NH}$. Only for relatively strong EMEP, the H-dibaryon appears as a bound state. However, even when the ground state in nuclear matter is $\Lambda\Lambda$, the continuum above the threshold is a mixed state of $\Lambda\Lambda$ and H-dibaryon, and there is a region where the H-dibaryon component becomes strong. To see this, the spectral function

$$S(E,P) = -\frac{1}{\pi}{\rm Im}[G_{\rm H}(E,P)] \tag{4·17}$$

is shown in Fig.4. Fig.4a corresponds to the case where the energies of the H-dibaryon exceed

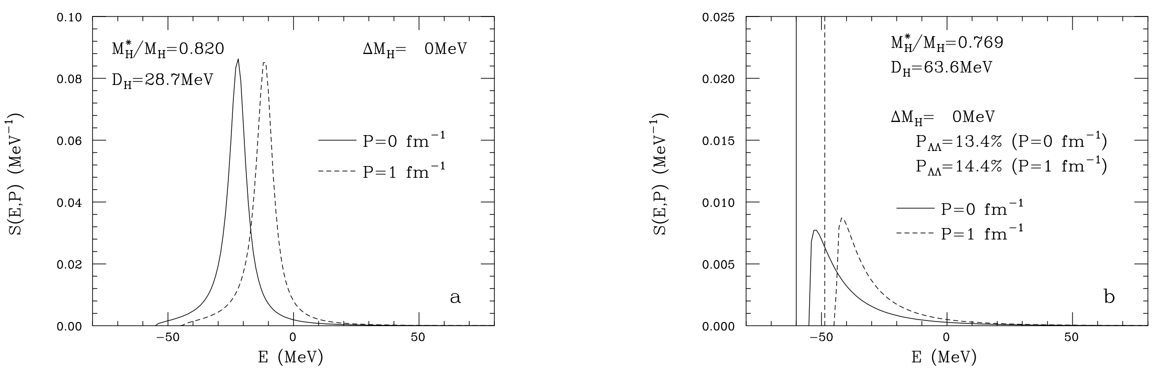

Fig. 4. (a) The spectral function, $S(E,P)$, at $P = 0$ and 1 fm^{-1} is shown for $M_{\rm H}^*/M_{\rm H} = 0.820$ and $D_{\rm H} = 28.7$ MeV. $\Delta M_{\rm H} = 0$. The solid (broken) line corresponds to $P = 0$ fm^{-1} ($P = 1$ fm^{-1}). (b) Same as (a) but for $M_{\rm H}^*/M_{\rm H} = 0.769$ and $D_{\rm H} = 63.6$ MeV. The amounts of the $\Lambda\Lambda$ admixture in bound states, $P_{\Lambda\Lambda}$, are also given. (Data in these figures are taken from Ref. 166).)

the $\Lambda\Lambda$ thresholds in nuclear matter, and shows that narrow ranges around peaks contain appreciable amounts of the H-dibaryon component. For comparison, shown in Fig.4b is the case where the H-dibaryon appears as the bound state, which is seen as the δ-function peak in the spectral function.

Though this work has been done for nuclear matter, it may give an implication to finite double hypernuclei. The situation for a finite nucleus, corresponding to the case where there is no H-bound state in nuclear matter, will be that the low-lying discrete states have the character of $\Lambda\Lambda$ bound states, but that some of excited-states may have strong admixture of the H-nuclear states. In the opposite case where H is bound below the $\Lambda\Lambda$-threshold in nuclear matter, it is expected that the ground states in finite nuclei with strangeness -2 will have the character of the H-nuclear states and the amounts of the mixing with $\Lambda\Lambda$ states are roughly given by the nuclear matter calculation.

§5. Interaction between H-dibaryons

In this section, we will review an investigation on the interaction between two H-dibaryons in Ref. 167). As the framework of the QCM is the same as in the work reviewed in the previous section, we do not repeat here.

Tamagaki suggested the possibility that H-matter appears at densities several times higher than normal nuclear density.[162] That work is based on an assumption that the CMI plays a key role in determining the properties of the H–H interaction. In this pioneering study, a simple H–H interaction model consisting of a hard-core potential plus a square well attractive potential outside the core was used, because no microscopic calculation of the H–H interaction was available at that time. In the work reviewed here,[167] more quantitative information on the H–H interaction is extracted from a microscopic calculation by employing the quark cluster model.

In Fig. 5 the binding energy of the HH system as a function of V_0^{HH} is shown. The HH scattering phase shift is shown in Fig. 6 as a function of the relative wave number k for several V_0^{HH}. The following results are obtained from this calculation.

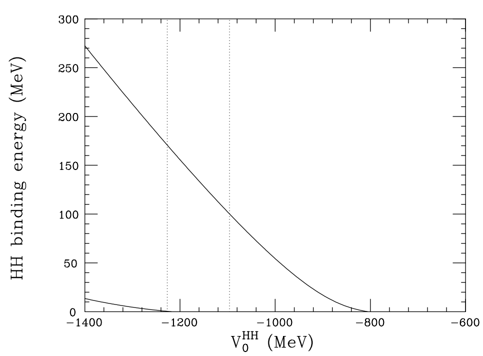

Fig. 5. The binding energy of the HH system as a function of the strength of EMEP V_0^{HH}. (Taken from Ref. 167).)

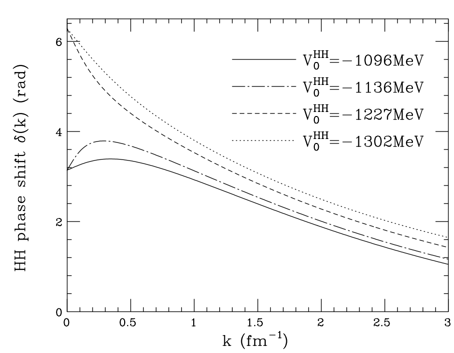

Fig. 6. The HH scattering phase shift δ as a function of the relative wave number k for several V_0^{HH}. (Taken from Ref. 167).)

(i) The interaction between H-dibaryons has a strongly attractive nature, and bears the deeply bound state. The binding energy in the ground state is 100∼170 MeV. Even a second bound state appears for $V_0^{\mathrm{HH}} \lesssim -1200$ MeV. (See Fig.5).

(ii) The main properties of the interaction between H-dibaryons can be characterized as a short-range repulsion due to Pauli blocking and the CMI, and a medium-range attraction due to flavor singlet scalar meson exchange (See Fig.6). The two H-dibaryons are bound with a separation of about $0.8 \sim 0.9$ fm due to the strong repulsive core. If we follow the discussion made in §4, the normalization kernel at zero separation is $\frac{25}{32}$, which indicates the effect of the Pauli blocking, and the CMI term bears a repulsive force as $\Theta_{\rm HH} - 2 \times \Theta_{\rm H} = 24 - 2 \times (-24) = 72$.

It is worth while noting that a bound state of two H-dibaryons is also obtained in the Skyrme model [168] although the binding energy of the "tetralambda" state ($E_{\rm B} = 15 \sim 20$ MeV) is rather smaller than the value obtained in the QCM. The attractive nature of the inter-H interaction is also implied in a study on H-dibaryon matter in the Skyrme model [169] following the method developed by Manton *et al.* [194]

The implications of this result for the occurrence of H-matter are as follows. In Tamagaki's discussion [162] that there is a possibility of a phase transition from neutron matter to H-matter at a density which is 6∼9 times greater than the normal nuclear density ρ_0, the interaction between H-dibaryons is through a hard core potential and an attractive square well potential outside the core. The depth of the attractive potential was assumed to be so weak that it can be treated as a perturbation. The depth of the square well potential has been determined so that the scattering length becomes zero for the first time, when the strength of the attractive potential is gradually increased. The pertinent strength parameter corresponding to the EMEP can be obtained from the scattering length calculated from S-matrix through the effective range theory. It is found to be $V_0^{\rm HH} = -638$ MeV. Thus the attraction used in Ref. 162) is much weaker than the one used in this QCM calculation, in which $V_0^{\rm HH}$ is typically $-1300 \sim -1100$ MeV. If the attractive H–H potential is indeed as strong as the EMEP employed in this calculation, the critical transition density beyond which H-matter formation is energetically favorable may be appreciably lower, although a more quantitative estimate is not simple because of the inapplicability of perturbation theory under a strong attractive potential as in the present calculation.

Some comments on implications of the existence of the H-dibaryon to neutron star properties are in order. In the framework of the Walecka model with the strength of the H-H interaction of the present calculation, it has been shown that H-matter is unstable against compression. [163] Therefore, if the central density of a neutron star exceeds the critical density for H-matter formation, the energetically favorable compression of H-matter could provide a scenario for the conversion of a neutron star into a strange quark star. [163]

Using a relativistic mean field theory, it is studied how H-dibaryon condensate affects the equation of state and the properties of neutron stars. [164] It is shown that, if the limiting neutron star mass is about the mass of the Hulse-Taylor pulsar ($1.44 M_\odot$), a condensate of H-dibaryons with their mass in the vacuum about 2.2 GeV and a moderately attractive potential in the medium could not be ruled out.

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