

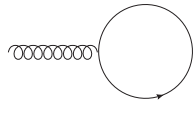
QCD and Precision Calculations

PREFIT School, DESY Hamburg, March 2020

Exercise: Colour Algebra

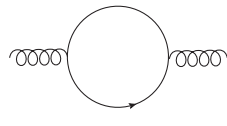
Calculate the colour factors for the following Feynman diagrams:

(1)



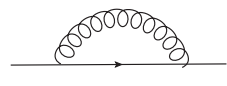
$$= \text{Trace}(t^a) = 0$$

(2)



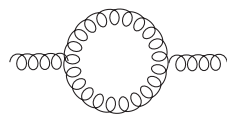
$$= t_{ij}^a t_{ji}^b = \text{Trace}(t^a t^b) = T_R \delta^{ab}$$

(3)



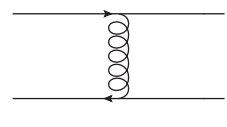
$$= t_{ij}^a t_{jk}^b \delta^{ab} = C_F \delta_{ik}$$

(4)



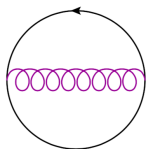
$$= f^{acd} f^{bcd} = C_A \delta^{ab}$$

(5)



$$= t_{ij}^a t_{lk}^a = T_R \left(\delta_{ik} \delta_{lj} - \frac{1}{N_c} \delta_{ij} \delta_{kl} \right)$$

(6)



$$= T_R \delta^{ab} \delta_{ab} = T_R (N_c^2 - 1)$$

$$= t_{ij}^a t_{jk}^a \delta_{ik} = C_F \delta_{ii} = C_F N_c$$

Therefore we derived $C_F = T_R \frac{N_c^2 - 1}{N_c}$.